WW SCATTERING USING ELECTROWEAK CHIRAL EFFECTIVE LAGRANGIANS

THE UNIVERSITY of MANCHESTER

A thesis submitted to the University of Manchester for the degree of MPhil in the Faculty of Science and Engineering

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Abstract

UNIVERSITY OF MANCHESTER

ABSTRACT OF THESIS submitted by Anahita Angela New for the Degree of MPhil and entitled **WW Scattering Using ElectroWeak Chiral Effective Lagrangians**

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We study the WW scattering process. Our main tool is the Electroweak Chiral Effective Lagrangians (EWCL) method. Using the Equivalence Theorem (ET), we investigate the symmetry breaking sector of the Standard Model via the study of the behaviour of the longitudinal polarisation states of the W gauge bosons. Several unitarisation procedures have been applied to remedy the bad high energy behaviour of our amplitudes. Our goal is to study the dynamics of the symmetry breaking and the resulting possibility of resonances. Through this procedure, we obtain bounds on the quartic couplings of the Electroweak Chiral Effective Lagrangian.

Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institution of learning.

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For their support and love, a huge thanks to my family as always.

A special thanks to Etera, my love, and my friends Steven, Suzinne, and Mike.

Never staying too long and always moving is something I learned from all these people, and a way to approach everything, including physics.

For showing me the possibilities, thanks to Jeff, my supervisor. And, I'm grateful to all those at Manchester University who made physics enjoyable.

For all the great people whose names are not mentioned, write them in yourself - you know you belong here. Thanks, thanks, thanks!

Dedication

To my friends

Chapter 1

Introduction

"*The question of the nature of the Higgs sector and the explicit mechanism of SU(2) × U(1) breaking is probably the most pressing open problem in the theory of elementary particles.*"[1]

This project is an investigation of WW scattering, which is a subprocess that will be investigated at the Large Hadron Collider (LHC). This particle accelerator is due to start running in 2005, based in Switzerland at CERN (Organisation Européenne pour la Recherche Nucléaire)[1]. The LHC will replace the Large Electron-Positron Collider (LEP) (for a review of the results obtained at the LEP, see [2]) in the test of the Standard Model and in the search for new physics.

The LHC is a proton-proton collider which will be able to obtain energies around 14 TeV (proton-proton). One of the processes which is possible, and the one which we are investigating, is that of two W's being radiated from quarks (one from each proton) which could then interact elastically. This would be an interaction in which the Higgs channel would be likely to show, and so an ample amount of data should be available to help investigate our claims. This study is to be considered in the more general framework of the proton-proton collision [3, 4]. Our approach is not so single-minded, however, as to look only for the Higgs. We would like to find a more global framework than the Minimal Standard Model from which we would deduce the existence of the Higgs. Such a framework exists and has been studied extensively, it is the Electroweak Chiral Effective Lagrangian approach (for a review of Effective Lagrangians, see [5]). This study is part of a bigger project whose goal is to constrain the couplings of this theory and also constrain the possible mass of the Higgs particle (see for example [6]). This work is based on the possibility that no Higgs may be detected. This would indicate some strong dynamical effects and lead us to imagine a different symmetry

[1]This acronym was once accurate as the longhand for it was previously "Conseil Européen pour la Recherche Nucléaire". This has since changed but the name CERN, by which most people refer to the site, has stuck.

breaking scenario. Electroweak Chiral Effective Lagrangians allow us to consider the most general spontaneous symmetry breaking. And we try to analyse the possible resonances using the techniques developed in this context.

1.1 The studied model: without Higgs

The theory we use to is that of the conventional Standard Model (SM) of particle physics, but without assuming the Higgs field *a priori*. This change is not without effect since the presence of the Higgs in the SM ensured unitarity, and hence renormalisability. The simpler theory containing a Higgs particle is called the Minimal Standard Model (MSM) and some details about it will be given in the next section.

An *Electroweak Chiral Effective Lagrangian* approach is used. This is a model-independent formalism, and as such, can reveal the possibilities for the new physics which we will encounter. It inherently demands this new physics to be able to deal with the infinite tower of higher dimensional operators that invariably occur in this approach. We make use of the Equivalence Theorem (ET). This enables the approximation of the amplitude of the longitudinal ElectroWeak (EW) bosons -neutral and charged- scattering $W_l W_l \to W_l W_l$ to that of the Goldstone Bosons (GB). These GB are equivalent to the pions, they are massless particles that occur whenever a continuous symmetry is broken. Usually, they are absorbed into the gauge bosons to make them massive. This is the scheme -Higgs mechanism- used to obtain massive bosons in the Standard Model while retaining its unitarity and renormalisability.

Our use of the Standard Model No Higgs (SMNH) destroys the pertubative unitarity of our theory: although the full scattering amplitude would be unitary, the ampitudes calculated order by order using Feynman diagrams are not. A non-unitary scattering amplitude can't have any physical meaning. We counter the problem by trying to restore unitarity in the calculated amplitudes. There exist various means of doing this that have appeared in the literature [7, 8, 9, 10]. The Padé Approximation, K Matrix and N/D Method form the set of procedures that we will investigate. These are all methods by which we may unitarise the amplitude in an approximate way.

After having restored unitarity in our calculations, we typically find resonances in the WW scattering process. They are of three kinds:

- *Scalar Resonance:* This corresponds to a null weak isospin $I = 0$ and a null angular momentum $J = 0$. It would be interpreted as a massive Higgs particle whose mass can be around 1 TeV.

- *Vector Resonance:* It is a resonance with weak isospin $I = 1$ and angular momentum $J = 1$. It is a vector resonance which can be compared to the technirho resonances which occurs in most techicolour models [27].

- *Doubly Charged Scalar:* This is a weak isospin $I = 2$ particle. As we will see later, this can not truly be a particle because it violates causality, so that sectors allowing for such a resonance will be labelled unphysical.

More precisely, we study the theory at the second order, there are 15 new terms in the lagrangian at this level of approximation but only 2 will be involved in the WW interaction process. The main practical goal of the present study is to be able to yield definite statements on the type of resonances and the masses to be expected for particular values of these quartic couplings, a_4 and a_5 of the model (or L_2 and L_1, correspondingly, depending on your notational preference). Then, in the case that there is indeed a strongly interacting Electroweak Symmetry Breaking Sector, and that LHC determines the value of the couplings to sufficient accuracy, we can make useful predictions based on that information and check the validity of the approach [11, 12, 13, 8, 15]. One can also constrain experimentally the other couplings of the theory in a more detailled study of the possible processes [16, 17, 18, 19], but this is not part of the present project.

There are some theoretical issues which need to be dealt with on the way to predicting the masses of the resonances. Unitarisation, for example, is necessary because we wish to consider the scattering of the longitudinal components of the W bosons at all energies, and more precisely at high energies $s \gg M_W^2$ where the Equivalence Theorem (ET) works best. This is non-trivial since we use chiral perturbation theory which realises an expansion in s/v^2 and works best at low energies. Therefore, one of our goals will be the comparison of different unitarisation procedures and their results, to see the consistency of the predictions arising from the use of several of them. In this way, we hope to show that our predictions will be more or less independent of the unitarisation procedure, and thus make them more credible.

1.2 About the Standard Model

I would like to here present a discussion of the Minimal Standard Model (MSM). This is a model which has been successful in its predictions. Unfortunately, there are a few important issues it doesn't tackle. For example, it doesn't explain the *hierarchy problem* that is the existence of very different mass scales. It also contains 19 parameters which must be input by hand. That is, there are nineteen numbers which must be prescribed with no fundamental idea behind these values having been incorporated into the model. For example, the neutrino mass is a free parameter. In the MSM, its value is set at zero. There is no rationale behind this choice, except maybe a concept of simplicity. We would have to have a very good reason to set it to another value with no evidence. If we have evidence to the contrary which is what we believe today using the results from SuperKamiokande [20], we must review our choices.

This is why scientists now take the view that it is not a complete nor a fundamental description of nature. Rather it should be considered as an effective theory - an approximation of the truth. That is, it will not be sufficient for the description of the physical world above some energy scale. This will become apparent, it is thought, as particle accelerators increase in energy. We have so far reached regimes of the order of 100 GeV, and this is increasing. The proton-antiproton collider Tevatron works at 2 Tev and LHC will start running in 2005 with 14 TeV (proton-proton).

The SM is a $SU(3) \otimes SU(2) \otimes U(1)$ gauged Quantum Field Theory (QFT) with a scalar Higgs sector that introduces Spontaneous Symmetry Breaking (SSB) into the model in order to create massive particles out of bosons. This is nicely explained by Veltmann in the CERN Yellow Report 97-05: “Particles are either massive at conception, or acquire mass by way of interacting with the Higgs field in the theory. All known particles in the MSM acquire their masses exclusively through the Higgs mechanism. The Higgs field functions as an ultraviolet cut-off, and without it the Yang-Mills interactions are non-renormalisable.”

There have been a plethora of attempts to better this model, to generalise it so that it will explain more than it currently does. Grand Unified Theories (GUTS) are one very well established example of these attempts. In GUTS, one uses a gauge group bigger than $SU(3) \otimes SU(2) \otimes U(1)$ like $SU(5)$, E_6,... in order to unify the different interactions. Higgs fields are then used to break the symmetry down to the one observed in reality. Usually, as in GUTS, the Higgs particle is assumed without any sense of where it comes from. The theory postulates them and doesn't create them. For a review of the different theories beyond the Standard Model, see [21].

In this thesis, the MSM, in the case of a Higgs particle much more massive than the W bosons, can be considered as an effective theory -a low energy approximation- of the ElectroWeak Chiral Effective Lagrangian that we employ.

In the second chapter of this thesis, we will describe the WW scattering process. We will introduce the approximation scheme, giving details on the Equivalence theorem, and conclude with the partial wave amplitudes of the scattering.

In the third chapter, we will discuss the problem of the unitarity of our partial wave amplitudes.

In the next chapters, we will use various unitarisation procedures in order to get some meaningful result. So chapter 4 will be about the *Padé approximation.* Chapter 5 will describe the *K-matrix* procedure and chapter 6 will tackle the *N/D method*, which is the best general approximation currently used to unitarise amplitudes.

Chapter 2

WW Scattering

2.1 Higgs mechanism in the Standard Model

The Higgs mechanism is a spontaneous symmetry breaking process. It describes the spontaneous breakdown of a continuous symmetry, the induced creation of massless modes [22], named Goldstone Bosons, and their absoption into the gauge bosons [23, 24] to give them masses.

More precisely, one introduces a scalar field, the Higgs field, whose potential's minimum will not for the null value of the field. We will thus have a non-zero expectation value of the field. We call $\mathcal{G}$ the initial gauge group and $\mathcal{H}$ the remaining gauge symmetry after the symmetry breaking. Quantum field theory is mainly the study of the perturbations around a stable configuration of the fields. Usually, we take the null configuration. This configuration respects all the symmetries of the initial Lagrangian. Here, we are going to choose a minimum of the scalar field. This explicit choice breaks the symmetry $\mathcal{G}$. What happens? At a first glance, we obtain some **massless modes** corresponding to the broken symmetries. They are called **Goldstone Bosons** (GB). They correspond to going from the chosen minimum around to the other minima. Therefore there is the same of GB than of generators of $\mathcal{G}/\mathcal{H}$. And, we have also genuine non-trivial perturbations around the chosen minimum, which are the remaining gauge particles.

Why would we introduce such a field? Having a non-zero expectation value of the field is going to allow us to introduce an effective mass to the gauge bosons. The initial $SU(2) \otimes U(1)$ gauge theory, the Standard Model without Higgs, is renormalisable for massless gauge particles. Introducing mass terms by hand would kill the renormalisability. Instead, we introduce the Higgs scalar field. The theory is renormisable and we get masses for the gauge bosons through the spontaneous symmetry breaking. What happens is that the GB are absorbed by the massless gauge bosons to give them mass: roughly, a massless spin 1 particle has only transverse degrees of freedom whereas a massive one has also a longitudinal degree of freedom, and the GB is going to become this longitudional degree of freedom. This is the reason the Equivalence Theorem works(see in a later section). It states that, at high energy, the scattering of the longitudinal component of the gauge boson is very well approximated by the one of the massless GB. We will not give any details on this mechanism, but we will give a quick overview of the symmetry breaking and mass generation in the ElectroWeak theory.

In the ElectroWeak theory, the gauge symmetry is $SU(2) \otimes U(1)$. The symmetry will be broken down to $U(1)_Q$ to give masses to the gauge bosons $W^{\pm}$ and Z^0. The remaining symmetry corresponds to the photon and describes the Electromagnetic theory. We start with a $SU(2)$ gauge field A_μ^a and a $U(1)$ gauge field B_μ, so that the scalar field kinematical term can be written as:

$$S_{\text{scalar}}[\phi] = \int \mathcal{D}_\mu \phi \mathcal{D}^\mu \phi \tag{2.1}$$

where $\mathcal{D}_\mu = \partial_\mu - igA_\mu^a T^a - ig'YB_\mu$ with T^a and Y respectively the $SU(2)$ generators (a half of the Pauli matrices) and $U(1)$ generators (the identity 2×2 matrix). We define the ElectroWeak mixing angle θ_W as the rotation angle between the fields (A^3, B) and the real particle fields (Z^0, A) which represent the Z gauge boson and the photon:

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix} \tag{2.2}$$

where

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

Defining the Electric charge generator as $Q = T^3 + Y$, the electric charge as

$$e = \frac{g}{\cos\theta_W}$$

and the W particles as

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2)$$

we can rewrite the covariant derivative as:

$$\mathcal{D}_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{\cos\theta_W} Z_\mu (T^3 - \sin^2\theta_W Q) - ieA_\mu Q$$

Choosing the expectation value of the field ϕ to be

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

we can read the masses of the gauge bosons by substituting $(0, v)$ to ϕ in equation (2.1). We get a massless particle A, the photon, and three massives gauge bosons:

$$M_W = g\frac{v}{2} \qquad M_Z = \frac{g}{\cos\theta_W}\frac{v}{2} \tag{2.3}$$

We define ρ as the quotient $(M_W/M_Z \cos\theta_W)^2$. The value of ρ in this simple symmetry breaking process is 1. This value has been checked experimentally to less than 1%. And thus, we want to keep this parameter to 1 theoretically. Nevertheless, it depends on the symmetry breaking process we used. Here, we have given an overview of the simplest one. However, more complex process might give a value different from 1. More generally, it is know that ρ is kept to 1 by a global $SU(2)$ symmetry named *custodial symmetry*. And this will be an important feature that our model will have to retain.

2.2 Electroweak Chiral Effective Lagrangians

We choose to work in the framework of Effective Field Theory. Its applications to particle physics is well described in [5]. But the main idea is already contained in these words of Weinberg [25]:

"*As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you're simply writing down the most general theory you could possibly write down. [...] The essential point in using effective field theory is you're not allowed to make any assumption of simplicity about the Lagrangian. Certainly you're not allowed to assume renormalizability.* "

We will apply this to the ElectroWeak theory retaining only the essential symmetries of the theory, writing down the most general Lagrangian. And then, we will compute the one-loop amplitude which will need to be renormisalised and unitarised to yield definite and meaningful results. We will describe the theory in this section and we will tackle the unitarity problem in the next one.

As we have chosen to take the Standard Model No Higgs (SMNH) approach, we have a useful theory of weak isospin and weak hypercharge which is nevertheless incomplete. The $\mathcal{L}_{SMNH}$ has corrections which are not renormalisable, ie. cannot be absorbed into any of the existing terms in the theory. New physics will be necessary. We are forced to include new terms to absorb the infinities, with an increasing number as we go to higher orders in corrections. There is a model independent formalism, the Electroweak Chiral Effective Lagrangian [5], in which we write the most general expression of the theory possible. Its form is constrained only by the need to retain the particle content of the SMNH and to satisfy the principle of gauge invariance.

$$\mathcal{L}_{E\chi} = \mathcal{L}_{SMNH} + \mathcal{L}_{\infty}^{NR} \tag{2.4}$$

By the nature of effective field theories, it is non-linear and also non-renormalisable since we include all possible terms consistent with the symmetries and thus have an infinite expansion. Although we have an infinite tower of higher dimensional operators surfacing through the $\mathcal{L}_{\infty}^{NR}$ term, these are suppressed by progressively increasing powers of E/Λ, and so we have only a finite number of terms at any given order (energy) in the lagrangian. Λ is the scale of new physics. It is a canonical scale which is needed on dimensional grounds as soon as one starts introducing higher-dimensional (non-renormalisable) operators.

In the SM, when one studies the scalar sector, one can use the decomposition of the scalar field Φ into Higgs particle $H(x)$ and Goldstone bosons represented by the matrix U:

$$U = exp\left(\frac{i\vec{\phi}\cdot\vec{\tau}}{v}\right) \tag{2.5}$$

where $\vec{\phi} = (\phi_1, \phi_2, \phi_3)$ are the GB, $\vec{\tau}$ are the Pauli spin matrices and v is the electroweak symmetry breaking scale of about 246 GeV. In the limit of a heavy Higgs of mass M_H, the Lagrangian can be written as:

$$\mathcal{L}(\Phi) = \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle + \mathcal{O}\left(\frac{v}{M_H}\right) \tag{2.6}$$

where D_μ is the covariant derivative involving the ElectroWeak gauge fields and where $\langle\rangle$ indicates the trace. In the unitary gauge $U \to 1$, this term reduces to mass terms for the boson W and Z. More generally, it carries no clues as to the physics of the symmetry breaking. To investigate effects to the symmetry breaking process, we will have to study higher order corrections which we will carry using chiral expansion techniques [5].

The Low Energy Theorem (LET) gives an expansion in derivatives (or powers of momentum) in the GB fields. In fact, Lorentz invariance requires only even powers to be present, and the unitarity of the matrix U, $U^\dagger U = 1$, requires that there be at minimum two derivatives for a non-trivial interaction [5]. Thus, we can write:

$$\mathcal{L} = \sum_n \mathcal{L}^{(2n)} \tag{2.7}$$

At the second order in powers of momentum, and assuming custodial symmetry, we find only the usual term [26]:

$$\mathcal{L}^{(2)} = \frac{v^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle \tag{2.8}$$

The next order, $\mathcal{O}(p^4)$, reveals the underlying theory, and hence the dynamics of the symmetry breaking, as it is implicitly encoded in the unknown parameters. In our study, there are only two terms with quartic couplings which are relevant at this order, with corresponding unknowns in the coefficients, a_4 and a_5. This simplification, from the original multiplicity of 15 terms [7], is mainly due to our assuming that custodial symmetry is preserved.

Then the next to leading order term is [5]:

$$\mathcal{L}^{(4)} = a_4[\langle D_\mu U D^\mu U^\dagger\rangle]^2 + a_5[\langle D_\mu U D^\nu U^\dagger\rangle]^2 \tag{2.9}$$

These first two orders, in their vertices, give us all the information we need to calculate the GB scattering at 1 loop.

2.3 Equivalence Theorem

We are considering the scattering of the longitudinal components of the W bosons $W_L W_L \to W_L W_L$ where $W_L = \{W_L^+, W_L^0, W_L^-\}$ in this study. The Equivalence Theorem states that the scattering amplitude, or equivalently the S matrix elements, of the longitudinal components of the massive Vector Bosons may be approximated to that of the unphysical Goldstone Bosons (GB) $\phi\phi \to \phi\phi$, with the proviso that this result is valid at energies $s \gg M_W^2$. This holds to all orders of perturbation theory [35]. More precisely, the ET states the equality of the amplitudes at first order [11]:

$$\mathcal{M}(W_L(P_1), W_L(P_2), \dots) = \mathcal{M}(w(p_1), w(p_2), \dots)_R + O(M_W/E_i) \tag{2.10}$$

A useful way of looking at ET is to examine its character, and to notice that the validity limit corresponds to the statement that the observables in the theory are continuous. As $M_W \to 0$, there is a complementary role reversal for the longitudinal components of the W bosons and the Goldstone modes, with the W_L transforming from physical states ($M_W \neq 0$) to unphysical ones ($M_W = 0$) whilst the GBs undergo the reverse metamorphosis [28].

The amplitude, of definite weak isospin I, once calculated, is of the form:

$$T_I(s,t) \sim \frac{s}{v^2} + A\left(\frac{s}{v^2}\right)^2 + O(s^3) \tag{2.11}$$

The linear term is universal, ie. model independent, as it only depends on the symmetry structure and scale - it provides no information on the dynamics of the symmetry breaking. The quadratic term is the lowest order one which is non-trivial, and the expansion at this point includes the parameters a_4 and a_5 which will define a particular model when their values are explicitly chosen.

2.4 Amplitude to One Loop

2.4.1 Scattering Amplitudes

All amplitudes can be calculated, using the following textbook formalism, from the $W^+W^- \to W^0W^0$ amplitude which will be noted $\mathcal{A}(s,t,u)$ with s, t, u the three usual

Mandelstam variables. More precisely, for the scattering process $W_L^a W_L^b \to W_L^c W_L^d$, we have the relation

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = \mathcal{A}(s,t,u)\delta^{ab}\delta^{cd} + \mathcal{A}(t,s,u)\delta^{ac}\delta^{bd} + \mathcal{A}(u,t,s)\delta^{ad}\delta^{bc} \quad (2.12)$$

This is all in terms of the weak isospin states $W^a, a = 1,2,3$. Transforming them into physical bosons $W_L^{\pm} = \frac{1}{\sqrt{2}}(W_L^1 \mp iW_L^2)$ and $Z_L = W_L^3$, we can make use of the superposition principle to obtain the full set of physical amplitudes [27]:

$$\mathcal{M}(W_L^+ W_L^- \to Z_L Z_L) = \mathcal{A}(s,t,u) \quad (2.13)$$

$$\mathcal{M}(Z_L Z_L \to W_L^+ W_L^-) = \mathcal{A}(s,t,u) \quad (2.14)$$

$$\mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-) = \mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) \quad (2.15)$$

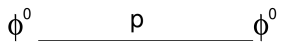
$$\mathcal{M}(Z_L Z_L \to Z_L Z_L) = \mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,t,s) \quad (2.16)$$

$$\mathcal{M}(W_L^{\pm} Z_L \to W_L^{\pm} Z_L) = \mathcal{A}(t,s,u) \quad (2.17)$$

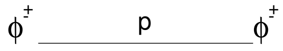
$$\mathcal{M}(W_L^{\pm} W_L^{\pm} \to W_L^{\pm} W_L^{\pm}) = \mathcal{A}(t,s,u) + \mathcal{A}(u,t,s) \quad (2.18)$$

2.4.2 Feynman Rules to One-Loop

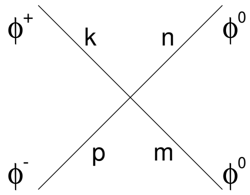
We apply the general techniques of quantum field theory to derive the Feynman rules ruling the scattering of the Goldstone Bosons ϕ, which approximate the WW scattering thanks to the Equivalence Theorem (ET). Only the two first order terms in the Lagrangian are involved in the one-loop calculations of the scattering, so we will derive only the corresponding Feynman rules. The first term, $\mathcal{L}^{(2)}$ (2.8), is composed of two derivatives. It yields the propagators of the GB and two 4-boson vertex rules. The second term, $\mathcal{L}^{(4)}$ (2.9), gives three 4-boson vertices. The rules are indicated in Fig.2.1 with all momenta in the 4-vertices directed toward the vertex.



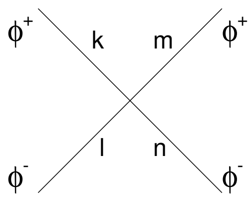
$$= \frac{1}{p^2}$$



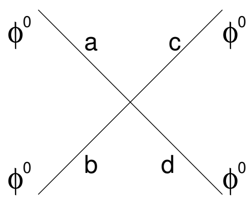
$$= \frac{1}{p^2}$$



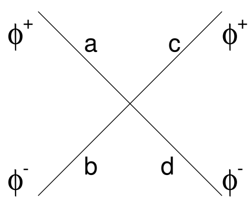
$$= -\frac{1}{6v^2}\left[(k_\mu - n_\mu)(m_\mu - p_\mu) + (k_\mu - m_\mu)(n_\mu - p_\mu) \right]$$



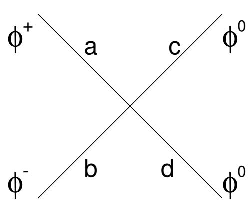
$$= -\frac{1}{24v^2}\left[(l_\mu - k_\mu)(n_\mu - m_\mu) + (l_\mu - m_\mu)(n_\mu - k_\mu) \right]$$



$$= \frac{4a_4}{v^4}(a_\mu b_\nu c_\mu d_\nu) + \frac{4a_5}{v^4}(a_\mu b_\mu c_\nu d_\nu)$$



$$= \frac{2a_4}{v^4}(a_\mu b_\nu c_\mu d_\nu + a_\mu b_\mu c_\nu d_\nu) + \frac{4a_5}{v^4}(a_\mu b_\mu c_\nu d_\nu)$$



$$= \frac{8a_4}{v^4}(a_\mu c_\mu b_\nu d_\nu) + \frac{8a_5}{v^4}(c_\mu d_\mu a_\nu b_\nu)$$

Figure 2.1: Feynman Rules to One-Loop

More precisely, to obtain these rules, we expand the field U in term of the field ϕ up to the order ϕ^4. We calculate the quadratic and quartic terms (cubic terms are absent). They respectively correspond to the propagator terms and the 4-vertices. For instance, in the case of $\mathcal{L}^{(2)}$, we get, with $\Phi = (\vec{\phi} \cdot \vec{\tau})$:

$$\mathcal{L}^{(2)} \approx \frac{1}{2}\langle \partial_\mu \Phi \partial^\mu \Phi \rangle + \frac{1}{12v^2}\langle (\Phi \overleftrightarrow{\partial_\mu} \Phi)(\Phi \overleftrightarrow{\partial^\mu} \Phi)\rangle \tag{2.19}$$

Then we compute these two terms using the explicit expression of the Pauli matrices τ_i. We get a formula directly involving the fields $\phi^0, \phi^\pm$. We go to momenta space and change the derivative of the fields into their momenta. Then the quadratic term will give the propagator of the fields after inverting it. And the second term will give the two 4-vertices described in Fig.2.1.

One can follow exactly the same procedure to get the three 4-vertices from $\mathcal{L}^{(4)}$. To the first non-trivial order, we get ϕ^4 terms involving four derivatives of the field. The corresponding rules are the last three vertices given in Fig.2.1 (these rules should be divided by a symmetry factor 8 to be exact).

2.4.3 Tree Level Amplitude

We calculate the amplitude of the process $W^+W^- \to W^0W^0$ (see Fig.2.2). $\mathcal{L}^{(2)}$ corresponds to the usual scattering processes encountered in the MSM. It contributes a vertex term proportional to $\frac{s}{v^2}$ and a one loop term, combining the two $\mathcal{L}^{(2)}$ vertices into the process $W^+W^- \to W^+W^- \to W^0W^0$, which is proportional to $\frac{s^2}{v^4}$.
The other term, $\mathcal{L}^{(4)}$, contributes also a tree level term proportional to $a_{4,5}\frac{s^2}{v^4}$.

The $\mathcal{L}^{(4)}$ tree level amplitude is:

$$\mathcal{A}(T) = \frac{a_4}{v^4}(a_\mu c_\mu b_\nu d_\nu) + \frac{a_5}{v^4}(c_\mu d_\mu a_\nu b_\nu) + \frac{a_4}{v^4}(a_\mu d_\mu b_\nu c_\nu) + \frac{a_5}{v^4}(c_\mu d_\mu a_\nu b_\nu) \tag{2.20}$$

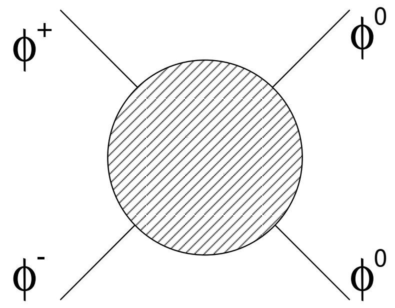


Figure 2.2: Amplitude we wish to calculate

giving, in terms of Mandelstam variables, the result:

$$\mathcal{A}(T) = \frac{1}{4v^4}[2a_5 s^2 + a_4(t^2 + u^2)] \tag{2.21}$$

2.4.4 One Loop Corrections

The amplitude for our loop diagram:

$$\begin{aligned}\int \frac{\mathrm{d}^4\mathrm{k}}{(2\pi)^4} \quad & -\frac{1}{6v^2}\left[(k_\mu + c_\mu)(k^\mu - a^\mu - b^\mu - d^\mu) + (k_\mu + d_\mu)(k^\mu - a^\mu - b^\mu - c^\mu)\right] \\ \times \quad & -\frac{1}{24v^2}\left[(b_\nu - a_\nu)(2k^\nu - a^\nu - b^\nu) + (b_\nu + k_\nu)(k^\nu - 2a^\nu - b^\nu)\right] \\ & \times \quad \frac{1}{k^2(a+b-k)^2}\end{aligned} \tag{2.22}$$

We use the identity:

$$\frac{1}{AB} = \int_0^1 \mathrm{dx} \frac{1}{[xA + (1-x)B]^2} \tag{2.23}$$

So, we find that our denominator can be altered to a new form:

$$\begin{aligned}\frac{1}{k^2(a+b-k)^2} &= \int_0^1 \mathrm{dx} \frac{1}{[x(a+b-k)^2 + (1-x)k^2]^2} \\ &= \int_0^1 \mathrm{dx} \frac{1}{[k^2 - 2(a+b)xk + (a+b)^2 x]^2}\end{aligned} \tag{2.24}$$

and our integral reads:

$$\int_0^1 \mathrm{dx} \int \frac{\mathrm{d}^4\mathrm{k}}{(2\pi)^4} \quad \frac{1}{144v^4}\left[(k_\mu + c_\mu)(k^\mu - a^\mu - b^\mu - d^\mu) + (k_\mu + d_\mu)(k^\mu - a^\mu - b^\mu - c^\mu)\right]$$
$$\times \quad \left[(b_\nu - a_\nu)(2k^\nu - a^\nu - b^\nu) + (b_\nu + k_\nu)(k^\nu - 2a^\nu - b^\nu)\right]$$
$$\times \quad \frac{1}{[k^2 - 2(a+b)xk + (a+b)^2 x]^2} \tag{2.25}$$

We have two identities concerning the momenta that we should bear in mind, namely the condition for massless pions:

$$a^2 = b^2 = c^2 = d^2 = 0 \tag{2.26}$$

and that for momentum conservation:

$$a + b = c + d \tag{2.27}$$

Using these, our integral simplifies to:

$$\int_0^1 \mathrm{dx} \int \frac{\mathrm{d}^4\mathrm{k}}{(2\pi)^4} \quad \frac{1}{144v^4} \frac{[kk + (c-a)k - b(a+c)][kk + (b-2a)2k - 2ab]}{[k^2 - 2(a+b)xk + (a+b)^2 x]^2} \tag{2.28}$$

This integral is infinite. We need to apply the so-called dimensional regularisation to be able to compute it. This means to formally change the dimension of space-time from 4 to n (see [1] for example). We then have techniques to compute formally the integrals and identify the divergence which appears when $n \to 4$. Removing these divergences is the renormlisation of the theory. Changing the dimension of space-time also modifies the diemsnion of the field ϕ which becomes

$$[\phi] = -\frac{n}{2} + 1 \tag{2.29}$$

This induces the introduction of an arbitrary energy scale μ in the Feynman rules to make sure that the Lagragian density has dimension $-n$. Therefore, the 4-vertex given by $\mathcal{L}^{(2)}$ gets a factor μ^{8-2n} (which of course reduces to 1 when $n \to 4$).

First, we want to compute the integral over k. This is done using the following identities which can be found in most quantum field theory books. They involve the Euler function Γ.

$$\int \frac{d^n k}{(k^2 + 2kq - m^2)^\alpha} = \frac{-\pi^{n/2}}{\Gamma(\alpha)(-q^2 - m^2)^{\alpha - n/2}} \tag{2.30}$$

$$\int \frac{d^n k \, k_\mu}{(k^2 + 2kq - m^2)^\alpha} = \frac{-\pi^{n/2}}{\Gamma(\alpha)(-q^2 - m^2)^{\alpha - n/2}} q_\mu \Gamma(\alpha - n/2) \tag{2.31}$$

$$\begin{aligned}\int \frac{d^n k \, k_\mu k_\nu}{(k^2 + 2kq - m^2)^\alpha} &= \frac{\pi^{n/2}}{\Gamma(\alpha)(-q^2 - m^2)^{\alpha - n/2}} (q_\mu q_\nu \Gamma(\alpha - n/2) \\ &\quad + \frac{1}{2} g_{\mu\nu}(-q^2 - m^2)\Gamma(\alpha - 1 - n/2))\end{aligned} \tag{2.32}$$

$$\begin{aligned}\int \frac{d^n k \, k_\mu k_\nu k_\lambda}{(k^2 + 2kq - m^2)^\alpha} &= \frac{\pi^{n/2}}{\Gamma(\alpha)(-q^2 - m^2)^{\alpha - n/2}} (-q_\mu q_\nu q_\lambda \Gamma(\alpha - n/2) \\ &\quad - \frac{1}{2}(g_{\mu\nu} q_\lambda + g_{\nu\lambda} q_\mu + g_{\lambda\mu} q_\nu) \\ &\quad \times (-q^2 - m^2)\Gamma(\alpha - 1 - n/2))\end{aligned} \tag{2.33}$$

$$\begin{aligned}\int \frac{d^n k \, k_\mu k_\nu k_\lambda k_\rho}{(k^2 + 2kq - m^2)^\alpha} &= \frac{\pi^{n/2}}{\Gamma(\alpha)(-q^2 - m^2)^{\alpha - n/2}} (q_\mu q_\nu q_\lambda q_\rho \Gamma(\alpha - n/2) \\ &\quad + \frac{1}{2}(q_\mu q_\nu g_{\lambda\rho} + \text{perm})(-q^2 - m^2)\Gamma(\alpha - 1 - n/2) \\ &\quad + \frac{1}{4}(g_{\mu\nu} g_{\lambda\rho} + \text{perm})(-q^2 - m^2)^2 \Gamma(\alpha - 2 - n/2))\end{aligned} \tag{2.34}$$

In our case, we note the following correspondences between our integral (2.28) and the above formulae:

$$k \leftarrow k$$

$$[-(a+b)x] \leftarrow q$$

$$[-(a+b)^2 x] \leftarrow m^2$$

$$2 \leftarrow \alpha$$

We then proceed to the integration over k. Keeping in mind that $s = -(a+b)^2$, the denominator term is always

$$\Gamma(2)(-s)^{2-\frac{n}{2}}(x^2+x)^{2-\frac{n}{2}} \tag{2.35}$$

To make sens of the power of s, we introduce once more the energy μ and use the following identity:

$$s^{-\epsilon} = \left(\frac{s}{\mu^2}\right)^{-\epsilon} \mu^{2\epsilon} = \mu^{2\epsilon}\left(1 - \epsilon \ln \frac{s}{\mu^2} + \dots\right) \tag{2.36}$$

The x term of the denominator is dimensionless and goes to 1 when n goes to 4, so it doesn't need to be taken in account when integrating over the variable x. After having proceeded to this further integration over x, we are finally ready to identify the divergences. They come in through the Γ function which has some poles at every negative integer. We use the following formulae giving the exact residue at the poles and the constant terms in function of the Euler-Marschoni constant γ:

$$\Gamma(2) = 1 \tag{2.37}$$

$$\Gamma(2-\frac{n}{2}) = \frac{2}{4-n} + \gamma + O(4-n) \tag{2.38}$$

$$\Gamma(1-\frac{n}{2}) = -\frac{2}{4-n} - 1 - \gamma \tag{2.39}$$

$$\Gamma(-\frac{n}{2}) = \frac{1}{4-n} + \frac{3}{4} + \frac{\gamma}{2} \tag{2.40}$$

which are valid for our case, where $n = 4$. Then finishing the one-loop calculations is straightforward and consist in a fastidious regrouping of the different terms to make the divergence clear in order to renormalise them. This is done in the next paragraph and the final amplitude is given then.

2.4.5 Renormalization

Renormalisation is the introduction of counter-terms in the Lagrangian in order to kill the infinities occuring when $n \to 4$ which is the physical limit. In our case, these counter-terms are simply going to be modifications of the couplings a_4 and a_5 since the divergent terms in the amplitude ar factors of s^2/v^4. Of course, we have a choice when absorbing the infinities: we can for example include constants like γ in them. It is actually our choice and the terms absorbed are [15]:

$$\lambda(a_4) = \frac{1}{6}\lambda \qquad \lambda(a_5) = \frac{1}{12}\lambda \tag{2.41}$$

avec

$$\lambda = \frac{\mu^{n-4}}{16\pi^2}\left[\frac{1}{n-4} - \frac{1}{2}(\log(4\pi) + \gamma + 1)\right] \tag{2.42}$$

Moreover, renormalization involves a dynamical change of the coupling constants depending on the chosen energy sclae μ. This gives [7]:

$$a_4(\mu) = a_4(\mu') - \frac{1}{16\pi^2}\frac{1}{12}\log\frac{\mu^2}{\mu'^2} \tag{2.43}$$

$$a_5(\mu) = a_5(\mu') - \frac{1}{16\pi^2}\frac{1}{24}\log\frac{\mu^2}{\mu'^2} \tag{2.44}$$

And the final amplitude to $\mathcal{O}(p^4)$ is [27]:

$$\begin{aligned}\mathcal{A}(s,t,u) &= \frac{s}{v^2}+\frac{1}{4v^4}[2a_5 s^2+a_4(t^2+u^2)]-\frac{1}{96\pi^2 v^4}[t(s+2t)\log(\frac{-t}{\mu^2}) \\ &+u(s+2u)\log(\frac{-u}{\mu^2})+3s^2\log(\frac{-s}{\mu^2})]\end{aligned} \tag{2.45}$$

One can easily check that the formula is independent of the choice of the energy scale μ both in the term involving the coupling constant and the one-loop renormalised term.

2.5 Partial Wave Amplitudes

The amplitude in the weak isospin basis, T_I, is projected into partial waves, t_{IJ}, with definite angular momentum (a.m.) J and isospin I, using the relation:

$$t_{IJ}=\frac{1}{64\pi}\int_{-1}^{1}\mathrm{d}(\cos\theta)\,P_J(\cos\theta)\,T_I(s,t) \tag{2.46}$$

The WW scattering system has I=0,1,2 as possible values. Bose symmetry further implies that only even J is allowed for I=0 and 2, while odd J is allowed for I=1, since we require 'I+J = even', and requiring particle stability means we are interested in lowest J at low energies as a matter of principle. There are only three possible weak isospin channels then, $t_{IJ}=t_{00},t_{11},t_{20}$ that we will deal with here.

These turn out to be [7]:

$$t_{00}^{(2)}=\frac{s}{16\pi v^2} \tag{2.47}$$

$$t_{00}^{(4)}=\frac{s^2}{64\pi v^4}\left[\frac{16(11a_5(\mu)+7a_4(\mu))}{3}+\frac{1}{16\pi^2}\left(\frac{101-50\log(s/\mu^2)}{9}+4i\pi\right)\right] \tag{2.48}$$

$$t_{11}^{(2)} = \frac{s}{96\pi v^2} \tag{2.49}$$

$$t_{11}^{(4)} = \frac{s^2}{96\pi v^4}\left[4(a_4(\mu) - 2a_5(\mu)) + \frac{1}{16\pi^2}\left(\frac{1}{9} + \frac{i\pi}{6}\right)\right] \tag{2.50}$$

$$t_{20}^{(2)} = \frac{-s}{32\pi v^2} \tag{2.51}$$

$$t_{20}^{(4)} = \frac{s^2}{64\pi v^4}\left[\frac{32(a_5(\mu) + 2a_4(\mu))}{3} + \frac{1}{16\pi^2}\left(\frac{273}{54} - \frac{20\log(s/\mu^2)}{9} + i\pi\right)\right] \tag{2.52}$$

First, one can easily check that these amplitudes are indepedent of the choice of the energy scale μ which is here only for dimensional reasons. A more important feature of these amplitudes is their non-unitarity. This can be easily checked and was expected since we used a perturbative approach to obtain them. Had they been calculated exactly they would have obeyed unitarity immediately, as does any physical amplitude. This does not cause a problem for us within the validity of the EWCL, $E \ll \Lambda$, since unitarity is only violated when $s \geq (4\pi v)^2$. However, since we wish to look at the theory at all energies, and this will mean exceeding the safe territory where the LET is valid, unitarity will be an issue.

Chapter 3

Unitarity

One of the most important features of the scattering matrix $\mathcal{S}$ relating in-states to out-states is its unitarity. This is a fundamental assumption in quantum field theory and translates the fact that the physics is deterministic and reversible: we don't lose any information during the scattering process. Moreover, as we are going to see in the following section, the unitarity puts constraints on the amplitudes calculated from the theory. Here, we are going to use only an approximate version of the unitarity principle: the elastic unitarity.

3.1 Elastic Unitarity

The matrix element of the scattering matrix $\mathcal{S}$ relating an in-particle state a to an out-particle state b is expressed in terms of the amplitude by the following relation:

$$\mathcal{S}_{ab} = \delta_{ab} + i(2\pi)^4\delta^{(4)}(\sum_{i\in(a)} p_i - \sum_{f\in(b)} p_f)\mathcal{A}_{ab} \tag{3.1}$$

where the $\delta^{(4)}$ implies the conservation of momentum.

The unitarity of the matrix $\mathcal{S}$ translates into $\mathcal{S}_{ac}\mathcal{S}^{\dagger}_{cb} = \mathcal{S}^{\dagger}_{ac}\mathcal{S}_{cb} = \delta_{ab}$, where the sum over the subscripts c is implicit. Therefore, the unitarity of the matrix $\mathcal{S}$ constrains the amplitude:

$$2\mathrm{Im}\mathcal{A}_{ab} = (2\pi)^4\delta^{(4)}(\sum_{i\in(a)} p_i - \sum_{f\in(b)} p_f)\sum_c \mathcal{A}_{ac}\mathcal{A}^{*}_{cb} \tag{3.2}$$

In our process, we are interested in $\phi\phi \to \phi\phi$, so that the in-particle state a is the same as the out-particle state b. Then one gets the optical theorem:

$$2\mathrm{Im}\mathcal{A}_{aa} = (2\pi)^4\delta^{(4)}(\sum_{i\in(a)} p_i - \sum_{f\in(c)} p_f)\sum_c \mathcal{A}_{ac}\mathcal{A}^{*}_{ca} \tag{3.3}$$

Elastic unitarity is an approximation in which we restrain ourselves to elastic scattering, and thus we take only the $c = a$ term from the sum over every possible state particle c. This is acceptable for $s \leq v^2$ [9]. This comes from the Low Energy Theorem. As an example, the simplest state after $\phi\phi$ is $\phi\phi\phi\phi$. Its contribution to the preceding equation at tree order is s^4/v^8 by L.E.T. which is much smaller than the s^2/v^4 contribution of the $c = \phi\phi$ term, and therefore can be neglected. More complex intermediate particle state have even higher order contributions which can be neglected the same way. More precise studies of the effect of the inelastic channels has been done and can be found in [32]. Nevertheless, the only thing to keep in mind in our framework is that the elastic channel is dominant.

To convert (3.4) to partial wave amplitudes, one should keep track of the angles, then rewriting (3.2) (still with $a = b$) with θ_1 the angle in the center of mass frame between the in-state a and the intermediate state c and θ_2 between c and the final state $b = a$.

$$\begin{aligned} -i\left[\mathcal{A}_{ac}(\cos\theta_1) \times \delta_{ca}\delta(\cos\theta_2 - 1) - \delta_{ac}\delta(\cos\theta_1 - 1) \times \mathcal{A}^*_{ca}(\cos\theta_2)\right] \\ = (2\pi)^4\delta^{(4)}(\textstyle\sum_{i\in(a)} p_i - \sum_{f\in(c)} p_f) \sum_c \mathcal{A}_{ac}(\cos\theta_1)\mathcal{A}^*_{ca}(\cos\theta_2) \end{aligned} \tag{3.4}$$

The decomposition formula of the amplitude into partial waves is the inverse, aside from a multiplicative factor, of (2.46):

$$\mathcal{A}(\cos\theta) = \sum_{l=0}^{\infty}(2l+1)a_l(t)P_l\left(1 + \frac{2s}{t}\right) = \sum_{l=0}^{\infty}(2l+1)a_l P_l(\cos\theta) \tag{3.5}$$

$\delta(\cos\theta - 1)$ is the distribution given by $a_l = 1/2$ for all l.

To project (3.4) into partial waves, we apply to it the following projector

$$\int_{-1}^{1} \mathrm{d}(\cos\theta_1)\, P_n(\cos\theta_1) \times \int_{-1}^{1} \mathrm{d}(\cos\theta_2)\, P_n(\cos\theta_2) \times$$

And using the orthonormality of the Legendre Polynomials

$$\int_{-1}^{1} \mathrm{d}x P_l(x) P_m(x) = \frac{2}{2l+1}\delta_{lm} \tag{3.6}$$

the left hand side of (3.4) reduces to

$$-i\left[a_n - a_n^*\right] = 2\text{Im}a_n \tag{3.7}$$

and its right hand side, when taking only the $c = a$ term, becomes

$$2 \times (a_n \times a_n^*) = 2|a_n|^2 \tag{3.8}$$

so that the optical theorem now reads:

$$\text{Im}a_n = |a_n|^2 \tag{3.9}$$

Here, in our case, $\mathcal{A}$ is T_I and a_l is t_{IJ}, so that

$$\text{Im}t_{IJ} = |t_{IJ}|^2 \tag{3.10}$$

Our amplitudes are given by pertubation series and don't satisfy the unitarity for the whole range of energy s. We need a unitarisation procedure to make them satisfy (3.10) at high s.

3.2 Unitarisation Procedures

What we have discovered is that our partial wave amplitudes leave us unequipped in our study of the WW scattering, as they violate the most basic property, unitarity, that any physical amplitude must comply with. To proceed we will need to either start anew with different amplitudes, or to manipulate the ones we have into a unitary form. We choose this second option, and the literature is rich with possibilities that we might consider using. In fact, we may invent as many unitarisation procedures as our enthusiasm allows. We have taken well established procedures in this study, and investigated them with reference to our problem. The ones which appeared the most popular and current were the Padé Approximation, K Matrix and N/D method. Comparison of levels of applicability, their sophistication and results was possible in this way.

This seemed an interesting methodology to pursue, as much work has been done in the literature comparing procedures. We intend to use the results obtained from various unitarisation procedures, and check how consistent the predictions appear to be. Mainly, this will be undertaken by comparison of resonance regions appearing in the a_5, a_4 parameter space. We hope that in this way, we will be able to find that unitarisation provides us with consistent results. Then we could commit ourselves to some level of accuracy as to the resonance structure to be expected at LHC. As we will see, unitarisation leads to resonances, or at least allows the existence of resonances to be ascertained, in a quite natural manner. In the future, we would hope to invent our own unitarisation procedures, and thus further strengthen our convictions that our predictions are robust.

Chapter 4

Padé Approximation

4.1 Theoretical Model

The Padé Approximation, otherwise known as the Inverse Amplitude Method (IAM), is one of the more simple unitarisation procedures, and is widely employed [7, 12, 13] in the literature. A partial wave amplitude, t_{IJ} is given as a series expansion in some parameter, in our case energy or momentum for the EChL, and so will be of the form:

$$t_{IJ} = t_{IJ}^{(2)}(s) + t_{IJ}^{(4)}(s) + O(s^3) \tag{4.1}$$

where the superscripts indicate powers in external momentum (or energy).

Elastic unitarity demands that the imaginary part of the amplitude is equal to the modulus squared of the amplitude, the Optical Theorem for partial wave amplitudes, ie.

$$\begin{aligned} Im(t_{IJ}) = |t_{IJ}|^2 \quad \Rightarrow & \quad Im(t_{IJ}^{-1}) = -1 \\ \Rightarrow & \quad t_{IJ}^{-1} = Re(t_{IJ}^{-1}) - i \end{aligned} \tag{4.2}$$

Using the approximation $(1+x)^{-1} \to 1 - x + h.o.$ gives

$$Re(t_{IJ}^{-1}) = Re\left(\frac{1}{t_{IJ}^{(2)}}\left(1 - \frac{t_{IJ}^{(4)}}{t_{IJ}^{(2)}}\right)\right) \tag{4.3}$$

The amplitude is then

$$t_{IJ} = \frac{1}{t_{IJ}^{(2)^{-1}}\left(1 - \frac{Re(t_{IJ}^{(4)})}{t_{IJ}^{(2)}}\right) - i} \tag{4.4}$$

since $t_{IJ}^{(2)}$ is real.

And then, we notice that our original partial wave amplitudes before unitarisation satisfy the following perturbative relation

$$Im(t_{IJ}^{(4)}) = |t_{IJ}^{(2)}|^2 \tag{4.5}$$

and so our amplitude can be approximated

$$t_{IJ} = \frac{t_{IJ}^{(2)}}{\left(1 - \frac{t_{IJ}^{(4)}(s)}{t_{IJ}^{(2)}(s)}\right)} \tag{4.6}$$

The fact that the Padé method matches the EWCL calculation to one-loop. This is in contrast to the K matrix say (see Sect.5) and indicates that the Padé method results may be quite valuable.

4.2 Application of the Padé Approximation

4.2.1 Scalar Sector

We apply the Padé method to the scalar case where I=0,J=0. Then one finds that the unitarised amplitude is

$$t_{00} = \frac{\frac{s}{16\pi v^2}}{1 - \frac{s}{4v^2}\left[\frac{16(11a_5(\mu)+7a_4(\mu))}{3} + \frac{1}{(4\pi)^2}\left(\frac{101-50\log(s/\mu^2)}{9} + 4i\pi\right)\right]} \tag{4.7}$$

The renormalisation scale μ can be set equal to $\sqrt{s}$ to remove the logarithm, so that we are able to be safe in our assumption that higher order corrections are small. We can do this without bias (in principle , though this is approximate when the amplitude expansion is truncated) since the chiral couplings are renormalised in such a way that we may choose the scale μ quite arbitrarily [12],

$$a_4(\mu) = a_4(\mu') - \frac{1}{16\pi^2}\frac{1}{12}\log\left(\frac{\mu^2}{\mu'^2}\right) \tag{4.8}$$

$$a_5(\mu) = a_5(\mu') - \frac{1}{16\pi^2}\frac{1}{24}\log\left(\frac{\mu^2}{\mu'^2}\right) \tag{4.9}$$

when this is translated to the form we have for t_{00} makes the amplitude invariant under a scale change.

The denominator of an amplitude for a Breit-Wigner resonance is of form

$$s - m^2 + im\Gamma \tag{4.10}$$

where m is the mass of the resonance, and Γ the width. From the real part of our denominator in the preceding equation one finds the mass

$$m_s{}^2 = \frac{12v^2}{16(11a_5(m_s) + 7a_4(m_s)) + \frac{101}{3(4\pi)^2}} \tag{4.11}$$

and the width

$$\Gamma_s = \frac{m_s^3}{16\pi v^2} \tag{4.12}$$

Then one can compute $\cot\delta_{IJ} = Re(t_{IJ}^{-1})$, and so calculate the phase δ_{IJ}:

$$\cot\delta_{00} = \frac{16\pi v^2}{s} - 4\pi\left[\frac{16(11a_5 + 7a_4)}{3} + \frac{101}{9(4\pi)^2}\right] \tag{4.13}$$

There will be a resonance when $\delta = \pi/2$, ie. when $\cot\delta = 0$. Therefore, the condition for a resonance occuring will be

$$11a_5 + 7a_4 > -\frac{3}{16}\frac{101}{9(4\pi)^2} \approx -0.0133 \tag{4.14}$$

One notices this is the condition for the square of the mass to be positive, ie. m_s is real. To each value of m_s^2 there corresponds a value of $11a_5+7a_4$, which gives a straight line in the (a_4, a_5) plane. Thus, giving limits to m_s^2 restricts the domain of a_4 and

a_5. The resonance would be effectively invisible to detectors if the width was greater than the mass of a particle, and so we employ this as a useful guideline to restrict our attention to detectable resonances. We use [7] the approximate experimental limit, $\Gamma_s \leq m_s$, where the resonance would be undetectable.

This physical limit gives

$$\frac{m_s^3}{16\pi v^2} \leq m_s \tag{4.15}$$

which implies:

$$11a_5 + 7a_4 > \frac{1}{16}\left(\frac{12}{16\pi} - \frac{101}{3(4\pi)^2}\right) \approx 0.0015 \tag{4.16}$$

4.2.2 Vector Sector

The I=J=1 sector gives a vector resonance, and can be computed in a similar manner, giving the mass and width as

$$m_v^2 = \frac{v^2}{4(a_4 - 2a_5) + \frac{1}{9(4\pi)^2}} \tag{4.17}$$

$$\Gamma_v = \frac{m_v^3}{96\pi v^2} \tag{4.18}$$

And the phase may be calculated from t_{11}

$$\cot \delta_{11} = \frac{96\pi v^2}{s} - 96\pi \left[4(a_4 - 2a_5) + \frac{1}{(4\pi)^2 9}\right] \tag{4.19}$$

Then, the condition for a vector resonance is

$$a_4 - 2a_5 > -\frac{1}{4 \times (4\pi)^2 \times 9} \approx -0.0002 \tag{4.20}$$

and the physical limit is

$$a_4 - 2a_5 > \frac{1}{4}\left(\frac{1}{96\pi} - \frac{1}{9(4\pi)^2}\right) \approx 0.0006 \tag{4.21}$$

4.2.3 Doubly Charged Scalar Sector

One can follow the same procedure for the $(I = 2, J = 0)$ partial wave amplitude which leads to

$$m_{20}^2 = -\frac{2v^2}{\frac{32(a_5+2a_4)}{3} + \frac{1}{16\pi^2}\frac{273}{54}} \tag{4.22}$$

$$\Gamma_{20} = -\frac{m_{20}^3}{32\pi v^2} \tag{4.23}$$

The phase is now

$$\cot\delta_{20} = -\frac{32\pi v^2}{s} - 16\pi\left[\frac{32(a_5+2a_4)}{3} + \frac{1}{16\pi^2}\frac{273}{54}\right] \tag{4.24}$$

And the condition for a resonance is

$$a_5 + 2a_4 < -\frac{3}{32}\frac{273}{54(4\pi)^2} \approx -0.0030 \tag{4.25}$$

with as physical limit

$$a_5 + 2a_4 < -\frac{3}{32}\left(\frac{1}{16\pi} + \frac{273}{54(4\pi)^2}\right) \approx -0.0048 \tag{4.26}$$

A problem arises in this doubly charged scalar channel because of the anomalous negative sign of $t_{20}^{(2)}$. Here, when there is a resonance, although the mass is real, the width Γ_{20} is negative. This corresponds to the phase δ crossing the $-\pi/2$ threshold, see Fig.4.3. This particle can't exist, it violates causality (due to a forbidden pole in the first Riemannian sheet [13]). This effect marks out a region which is excluded from possessing a true resonance structure, ie. an unphysical region.

Nevertheless, in this case, the part of the "resonance" region bounded by the physical and theoretical limits, i.e the saturation region, is not theoretically excluded.

4.3 Padé Resonance Spectrum

4.3.1 Plots of Phase Shifts: Illustrative Examples

We can plot the phase shifts, δ_{IJ} we calculated in (4.13), (4.19) and (4.24) against the centre of mass energy. When δ reaches the critical value of $\frac{\pi}{2}$, a resonance has occured, with its mass corresponding to the value of s at that point. It is possible that no resonance occurs, but for interest, for each channel, the a_4 and a_5 parameters have been suitably chosen so that $\frac{\pi}{2}$ is reached. Notice the peculiar behaviour of the $(2,0)$ channel, where the phase shift increases in a negative sense, and so violates causality. The other two channels are true resonances.

Note that the plots, in figures (4.1)(4.2)(4.3) have been done on a π periodic vertical scale, and so this is why there appears to be a discontinuity where there isn't one. This allows us to see the mass of the resonance which occurs, since the curve 'jumps' at exactly $\delta = \pi/2$.

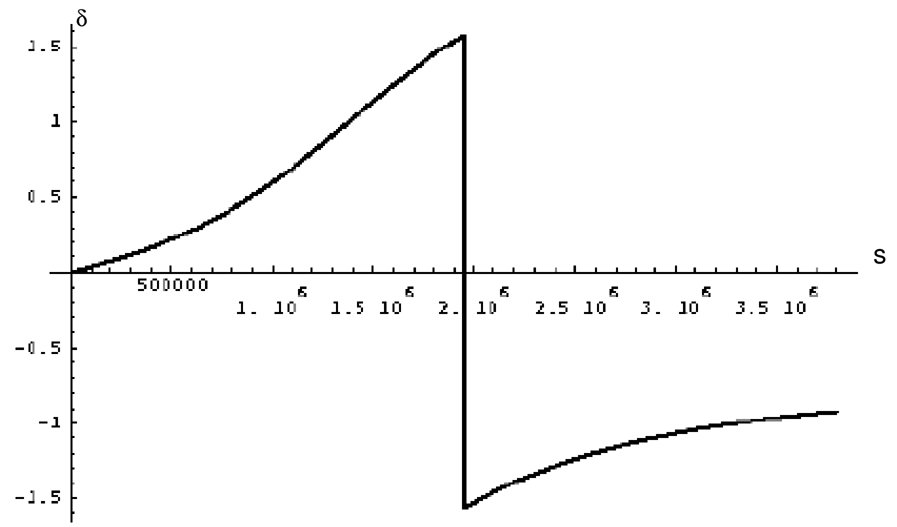


Figure 4.1: Scalar: δ_{00} plotted for $11a_5 + 7a_4 = 0.01$

4.3.2 Plots of $(a_5,\ a_4)$ Parameter Space

In figures (4.4) and (4.5), V stands for the Vector resonance regions of parameter space, and S and T correspondingly for the Scalar and Doubly Charged Scalar ones - with the lower case in each instance indicating the corresponding saturation regions. The saturation regions are those bounded by the theoretical boundary for the resonance region and the physical boundary (where $\Gamma \leq m$).

Notice the small triangular region in Fig.4.5 bypassed by all three resonance regions, indicated in the diagram by the arrow, that has no resonances whatsoever. In fact, experimentally, one would most likely not detect any resonance in the saturation regions so that the no-resonance region is substantially bigger that the tiny region indicated on Fig.4.5. It would certainly be disconcerting if LHC should discover that a_4 and a_5 lie unequivocally in this section of parameter space.

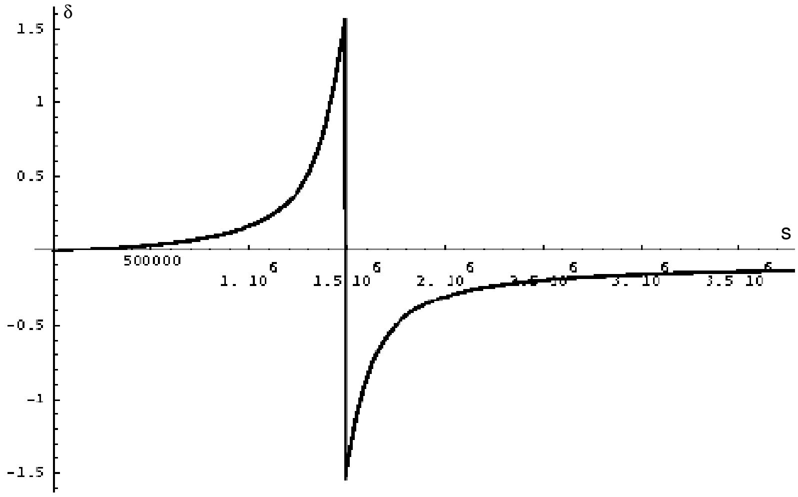


Figure 4.2: Vector: δ_{11} plotted for $a_4 - 2a_5 = 0.01$

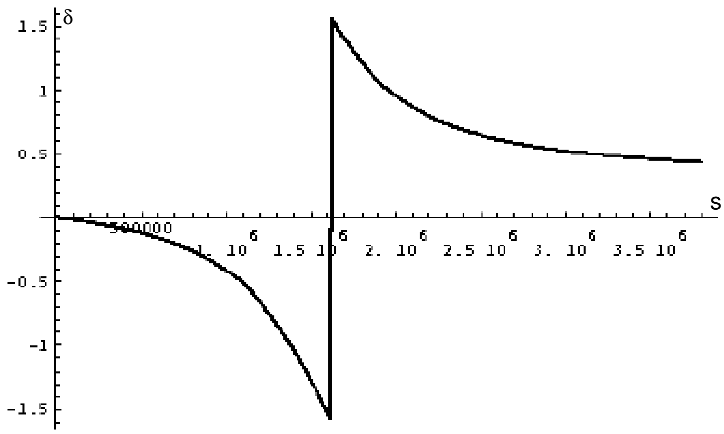


Figure 4.3: Doubly Charged Scalar: δ_{20} plotted for $a_5 + 2a_4 = -0.01$

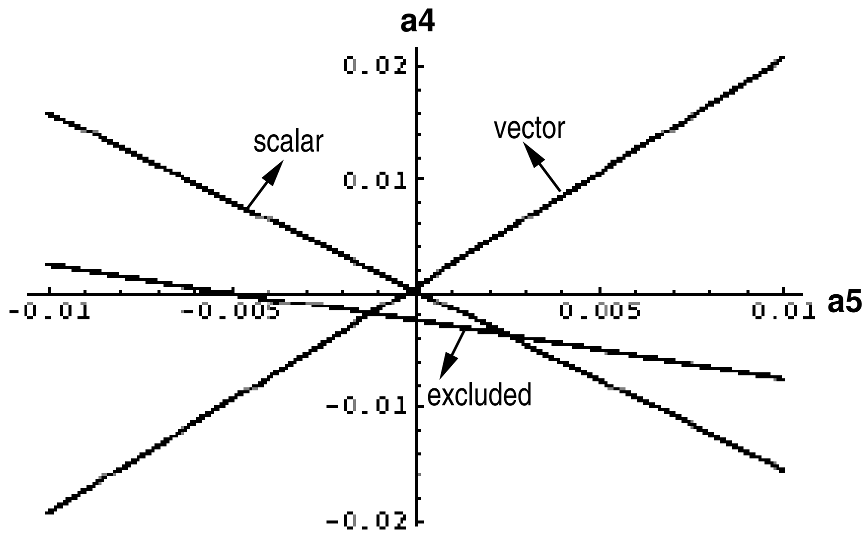


Figure 4.4: Padé Method Resonance Structure: Rough Idea - Physical Limits

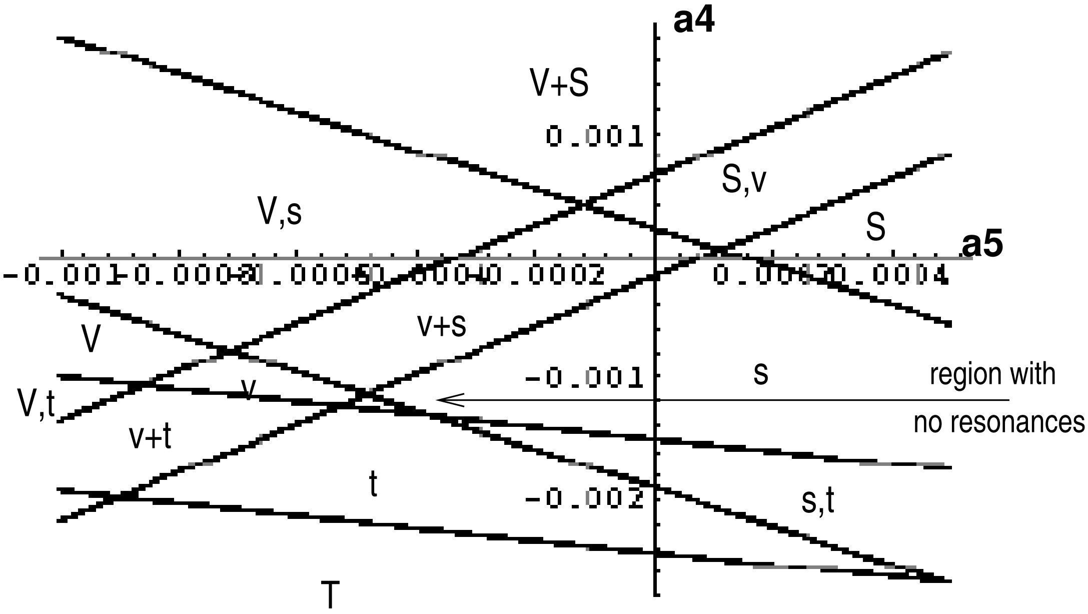


Figure 4.5: Padé Method Resonance Structure: Detailed

Chapter 5

K matrix

5.1 Theoretical Model

An ansatz is taken, firstly, such that the K matrix is defined in terms of the S matrix as:

$$S = \frac{1 + \frac{iK}{2}}{1 - \frac{iK}{2}} \tag{5.1}$$

The unitarity of the S matrix, $S^\dagger S = 1$ is equivalent to imposing hermiticity on K, $K^\dagger = K$, since:

$$\frac{1 + i\frac{(K-K^\dagger)}{2} + \frac{K^\dagger K}{4}}{1 - i\frac{(K^\dagger - K)}{2} + \frac{K^\dagger K}{4}} = 1 \tag{5.2}$$

$$\Rightarrow K - K^\dagger = K^\dagger - K \tag{5.3}$$

In terms of the transition matrix,

$$S = 1 + 2iT \tag{5.4}$$

$$\Rightarrow 2T = \frac{K}{1 - \frac{iK}{2}} \tag{5.5}$$

$$\Rightarrow T = \frac{K^{'}}{1 - iK^{'}} \tag{5.6}$$

This amplitude clearly satisfies elastic unitarity, ie. equation (3.10). Notice that our normalisation of the transition matrix has been chosen for this exact purpose. If K is a real number, for instance, a unitary scattering amplitude can surely be calculated. The K matrix prescription has been widely employed in the literature, see for example [27, 9, 14].

5.2 Application of the K Matrix

We choose first to unitarise the tree level amplitude (since it is automatically real) for the t_{00} channel. As will be seen, this method draws similar results, and identical conclusions, for the other channels under our consideration.

Our unitarised amplitude is:

$$t_{00} = \frac{\frac{s}{16\pi v^2}}{1 - \frac{is}{16\pi v^2}} \tag{5.7}$$

We know that $Re(t^{-1}) = \cot\delta$, which implies

$$\cot\delta_{00} = \frac{16\pi v^2}{s} \tag{5.8}$$

This has the correct behaviour at zero energy, since:

$$s \to 0 \Rightarrow \cot\delta \to \infty \Rightarrow \delta = 0$$

as expected.

And, at the other end of the energy spectrum,

$$s \to \infty \Rightarrow \cot\delta \to 0 \Rightarrow \delta \to \frac{\pi}{2}$$

which spells out that we only tend to a resonance in the limit $s \to \infty$.

Notice the main failing of this procedure was its complete exclusion of any information on the symmetry breaking dynamics, due to its not including a_4 and a_5. We now attempt to rectify this by trying a new approximation, using as our K matrix:

$$K_{00} = Re[t_{00}^{(2)} + t_{00}^{(4)}] \tag{5.9}$$

where the amplitudes $t_{00}^{(2,4)}$ are the partial waves amplitudes given in equations (2.47) and (2.48). From this, we can easily deduce the phase. Indeed, using (5.6), we see that

$$\cot\delta = \mathrm{Re}(t^{-1}) = \frac{1}{K} \quad \Rightarrow \quad \tan\delta = K \tag{5.10}$$

So, that now

$$\tan\delta_{00} = \frac{s}{16\pi v^2}\left[1 + \frac{s}{4v^2}\left[\frac{16(11a_5 + 7a_4)}{3} + \frac{1}{4\pi^2}\left(\frac{101 - 50\log(\frac{s}{\mu^2})}{9}\right)\right]\right] \tag{5.11}$$

This has the right behaviour at the zero end of the energy spectrum, and goes to $\pm\pi/2$ as the energy goes to ∞. This could be a problem, as the negative case would indicate a causality violation, but this would only be in the limit - and so never actually realised. However, this has the added advantage of the previous trial that it explicitly includes a_4 and a_5. It yields the same conclusions, in fact, as for the previous attempt! The K matrix prescription unitarises without problem, but does not produce resonances. The main drawback of this method is that the K-matrix prescription does not match the chiral perturbation theory calculation except at tree level and thus, in a way, it doesn't take in account the right dynamics.

Chapter 6

N/D method

6.1 Theoretical Model

The essence of the N/D method, N for Numerator and D for Denominator, lies in transforming an amplitude into a quotient of two very specifically defined functions. The right hand (or unitarity) cuts are placed wholly into the denominator function, D.

Then analyticity and unitarity demand the following relations are satisfied [15]:

$$ImD(s) = NImT(s)^{-1} = -N \qquad s > 0 \tag{6.1}$$

$$ImD(s) = 0 \qquad s < 0 \tag{6.2}$$

$$\tag{6.3}$$

$$ImN(s) = DImT(s) \qquad s < 0 \tag{6.4}$$

$$ImN(s) = 0 \qquad s > 0 \tag{6.5}$$

where

$$T(s) = \frac{N(s)}{D(s)} \tag{6.6}$$

This method possesses origins in the idea that [29] a function $F(s)$ can be decomposed into a quotient of $N(s)$ and $D(s)$ by using Omnès function $\mathcal{D}(s)$, and using elastic unitarity. Then, the simple fact that crossing symmetry may be used to switch back and forth between the two functions in an approximate manner means that we have two independent methods at our disposal - and so have a self-consistent method. Note that the full N/D method, as opposed to partial versions such as 'simple N/D', is often referred to as 'self-consistent N/D'. There are several strategems, aside from the one used here, see [15], in the literature, see for example [30, 31].

6.2 Application of the N/D Method

6.2.1 N/D: Scalar Sector

We transform the imaginary part of the t_{00} amplitude, obtained by summing equations (2.47) and (2.48), using the identity $\log(-x) = \log(x) - i\pi$ in order to arrive at a suitable form for our purposes:

$$t_{00} = \frac{1}{16\pi}\left[\frac{s}{v^2} + \frac{s^2}{4v^4(4\pi)^2}\left\{\left(\frac{(4\pi)^2 16A}{3} + \frac{101}{9}\right) - \frac{14}{9}\log\left(\frac{s}{\mu^2}\right) - 4\log\left(\frac{-s}{\mu^2}\right)\right\}\right] \tag{6.7}$$

where $A = 11a_5(\mu) + 7a_4(\mu)$.

Following Oller's methodology, we sort left and right hand cuts into the appropriate function, using:

$$N(s) = \frac{1}{16\pi}(1 + \delta N(s)) \tag{6.8}$$

$$D(s) = K(s)^{-1} + g(s)(1 + \delta N(s)) \tag{6.9}$$

where $\delta N(s)$ contais the left hand cut and $g(s)$ contains the right hand cut, and the parameter a is arbitrary and of order one.

Notice that the purpose of the factor $(1 + \delta N(s))$ multiplying $g(s)$ in the denominator appears to be so that our amplitude satisfies elastic unitarity. In other words it forces the inverse amplitude, T^{-1}, to possess an imaginary part equal to -1.

We proceed by writing our versions of these functions, based on calculations in [15, 10]: Noticing that our coefficient of $\log\left(\frac{-s}{\mu^2}\right)$ matches that of Oller's, we can use the same $g(s)$, namely:

$$g(s) = \frac{1}{16\pi^2}\left(log\left(\frac{-s}{\mu^2}\right) - a\right) \tag{6.10}$$

a is an arbitrary constant, more precisely the constant term beside all the pole terms in the denominator D [10]. It is somewhat similar to the arbitrary constant involved in the renormalisation of the couplings a_4, a_5. Nevertheless, it gains a physical effect through our procedure modifying the resonances and their mass. Thus, a principle or a procedure should be found to be fix (though it could depend on the energy scale). In the case we don't find one, one could finetune it so that the results of the N/D method fit the ones obtained from the Padé approximation scheme in the regular situations (low s).

μ is also an arbitrary constant. It is an energy scale independent from the choice of a. I have chosen $\mu \approx 200$GeV for the calculations and plots. It seems a practical energy scale, since the expected resonance masses can be expressed in units of μ (from 4 μ to 20 μ) in the regular situations.

Now, restricting ourselves to the 1 loop chiral calculation, we find the appropriate K function is:

$$K(s) = \frac{s}{v^2} - \frac{s^2}{4v^4(4\pi)^2}\left\{4a - \left(\frac{(4\pi)^2 16A}{3} + \frac{101}{9}\right)\right\} \tag{6.11}$$

With this information, we are in a position to unitarise our amplitude:

$$t_{00} = \frac{\frac{1}{16\pi}}{[K(1+\delta N)]^{-1} + g} \tag{6.12}$$

$$= \frac{\frac{1}{16\pi}X}{1 + \frac{1}{(4\pi)^2}\left(\log\left(\frac{-s}{\mu^2}\right) - a\right)X} \tag{6.13}$$

where

$$X = K(1+\delta N) = \frac{s}{v^2} - \frac{s^2}{4v^4(4\pi)^2}\left\{4a - \frac{(4\pi)^2 16A}{3} - \frac{101}{9} + \frac{14}{9}\log(\frac{s}{\mu^2})\right\} \tag{6.14}$$

This satisfies elastic unitarity, which can be checked by remembering to use the trick we used to convert our imaginary part before. We then can extract the phase shift:

$$\cot\delta_{00} = Re\left(\frac{\left(\log\left(\frac{-s}{\mu^2}\right) - a\right)}{\pi} + \frac{16\pi}{X}\right) \tag{6.15}$$

Now, we can plot the phase shifts against energy, and look at the behaviour with the parameter a set to various values, see figures (6.1), (6.2), and (6.3). It is apparent, after looking at the shape of the graphs that a serves to increase the vertical spread of the phase, and so can make the difference, quite naturally, between a case where there is a resonance and where not. The value we take for the $11a_5 + 7a_4$ combination can affect the character of our resonance. In fact, with Fig.6.3, it is difficult to know how to interpret such behaviour of the phase shift, since if a was increased sufficiently might incorporate what we would ordinarily term a true resonance.

Moreover, one should notice that for such an "ordinary" value of a as 1.5 and for values of $11a_5 + 7a_4$ as -0.01 or less, one gets a fake resonance: the phase goes through $-\pi/2$. This mean that the corresponding region $(a4, a5)$ is excluded. This is different from what happened in the case of the Padé approximation in which the corresponding region was simply discarded as not containing any scalar resonance. The same phenomena is expected for the vector resonance. However, such an inversion could lead to a surprise in the doubly-charged sector if one gets a true resonance. This is an interesting phenomena one could study. This would mean we get a very different picture from the one obtained using the Padé approximation. Furthermore, I think we should look more carefully at the consistency of the N/D method, more precisely to its domain of validity i.e the range of physically acceptable ennergies s. As an example, in the case $a = 1.5$ and $11a_5 + 7a_4 = 0.01$, we get a true resonance around $s \approx 3.10^6$MeV (Fig.6.2). Nevertheless, if we follow the plot up to very high energies as $s \approx 10^{11}$MeV, we see that we also get a fake resonance which would mean we should exclude this value of $11a_5 + 7a_4$. This happens in the whole region of values in which we expect to find resonances from the Padé method results. So further study should be done on the precise domain of validity of this unitarisation method.

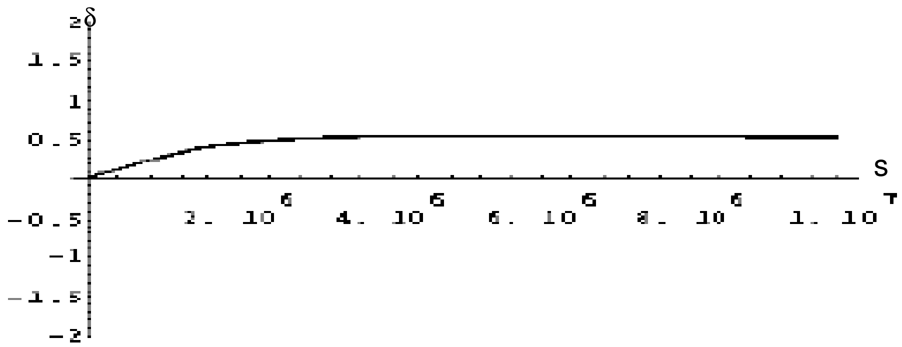


Figure 6.1: Phase shift for a = 0 with $11a_5 + 7a_4 = 0.01$

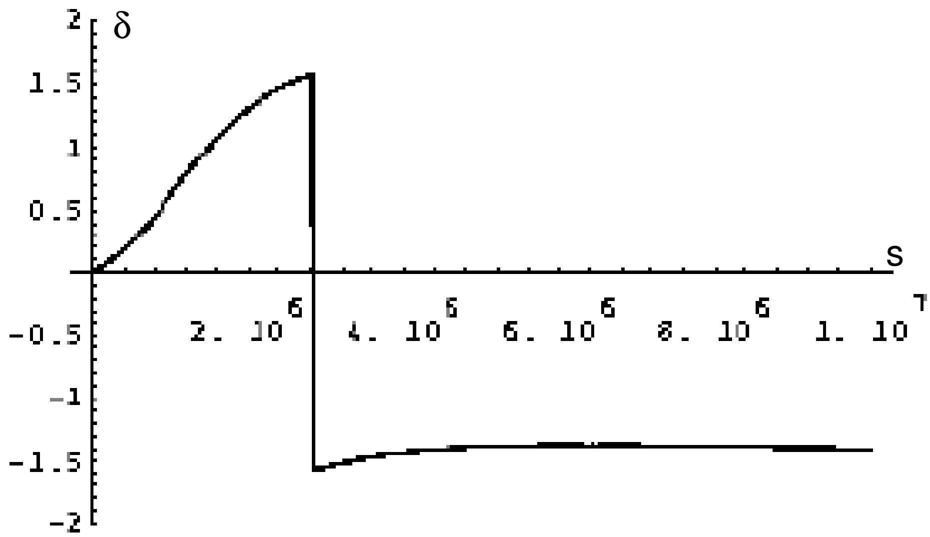


Figure 6.2: Phase shift for a = 1.5 with $11a_5 + 7a_4 = 0.01$

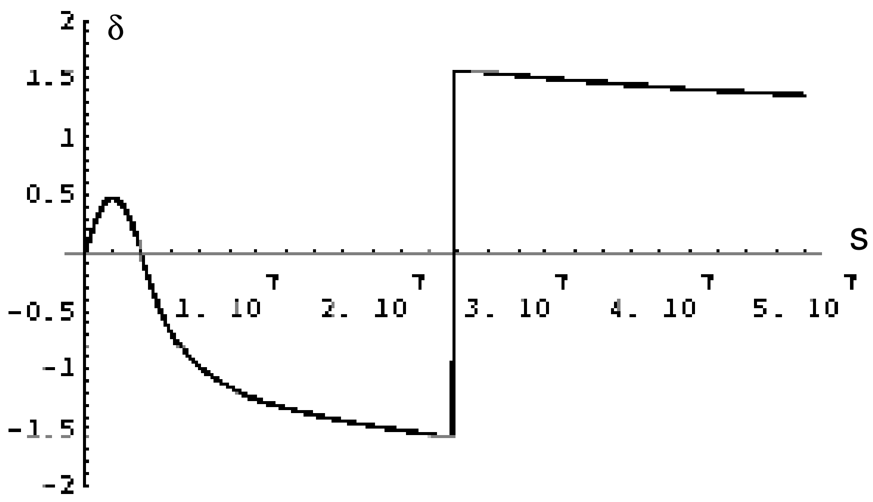


Figure 6.3: Phase shift for a = 1.5 with $11a_5$ + $7a_4$ = -0.01

6.2.2 N/D: Vector Sector

We transform the imaginary part of the t_{11} amplitude, obtained by summing equations (2.49) and (2.50) as previously done for the scalar sector.

$$t_{11} = \frac{1}{96\pi}\left[\frac{s}{v^2} + \frac{s^2}{96\pi^2 v^4}\left\{\left(4(a_4 - 2a_5)96\pi^2 + \left(\frac{2}{3} + \log(s/\mu^2) - \log(-s/\mu^2)\right)\right)\right\}\right] \tag{6.16}$$

Then,

$$g(s) = \frac{1}{96\pi^2}\left(log\left(\frac{-s}{\mu^2}\right) - a\right) \tag{6.17}$$

and,

$$K(s) = \frac{s}{v^2} - \frac{s^2}{96\pi^2 v^4}\left\{a - \left(96\pi^2 4(a_4 - 2a_5) + \frac{2}{3}\right)\right\} \tag{6.18}$$

We now unitarise:

$$t_{11} = \frac{\frac{1}{96\pi}}{[K(1+\delta N)]^{-1} + g} \tag{6.19}$$

$$= \frac{\frac{1}{96\pi}X}{1+\frac{1}{96\pi^2}\left(\log\left(\frac{-s}{\mu^2}\right)-a\right)X} \tag{6.20}$$

where

$$X = \frac{s}{v^2} - \frac{s^2}{96\pi^2 v^4}\left\{a - 96\pi^2 4(a_4 - 2a_5) - \frac{2}{3} - \log(\frac{s}{\mu^2})\right\} \tag{6.21}$$

The phase shift is now:

$$\delta_{11} = Re\left(\frac{\left(\log\left(\frac{-s}{\mu^2}\right)-a\right)}{\pi} + \frac{96\pi}{X}\right) \tag{6.22}$$

6.2.3 N/D: Doubly Charged Sector

The amplitude is derived from equations (2.51) and (2.52) to be:

$$t_{20} = -\frac{s}{32\pi v^2} + \frac{s^2}{64\pi v^4}\left[\frac{32(a_5(\mu)+2a_4(\mu))}{3} + \frac{1}{16\pi^2}(\frac{273}{54} - \frac{11\log(s/\mu^2))}{9} - \log(-s/\mu^2)\right] \tag{6.23}$$

And,

$$g(s) = -\frac{1}{32\pi}(\log(-s/\mu^2) - a) \tag{6.24}$$

with

$$K(s) = \frac{s}{v^2} + \frac{s^2}{32\pi^2 v^4}\left[a - \frac{273}{54} - \frac{32(4\pi)^2(a_5(\mu)+2a_4(\mu))}{3}\right] \tag{6.25}$$

and

$$X = K(1+\delta N) = \frac{s}{v^2} + \frac{s^2}{32\pi^2 v^4}\left[a - \frac{273}{54} - \frac{32(4\pi)^2(a_5(\mu)+2a_4(\mu))}{3} + \frac{11\log(s/\mu^2)}{9}\right] \tag{6.26}$$

giving a phase shift

$$\cot\delta_{20} = \left(\frac{-32\pi}{X} + \frac{1}{\pi}(\log(s/\mu^2) - a)\right) \tag{6.27}$$

6.3 The Overall N/D Picture

This has not been a comprehensive study - the vector and doubly charged scalar channels still require analysis. There also remain many questions on the N/D method and how to interpret its results. There is also the question of the a parameter and its significance. Using $a = 1.5$ allowed us to obtain our holy grail, a resonance, but is it a useful prediction? In the literature, a is typically of order one, so setting it to 1.5 seems reasonable. More strikingly, there is also the question of how the resonance regions appear in the a_4,a_5 plane. To find some way of finding bounds and to then plot them for comparison with other methods, specifically Padé presently, is crucial. However, if you notice the two figures which have a set to 1.5, and their values for $11a_5 + 7a_4$, and corresponding results - there does seem to be agreement with the Padé results. So, at least, this value of a looks promising. Still, before taking the results of the N/D method more seriously, one should be able to interpret the curious phenomenom which leads to a causality violating resonance at high energy at the same time as getting the expected resonance at a few TeV.

Chapter 7

Conclusion

This thesis has presented the framework of the ElectroWeak Chiral Effective Lagrangians in order to study the WW scattering and the ElectroWeak symmetry breaking in a broader context that the Minimal Standard Model. We have also introduced the current unitarisation procedures used in the literature to extract physical scattering amplitudes from a perturbative expansion: the K-matrix method, the Padé approximation scheme and the N/D method. This has lead us to the possible existence of some resonances (scalar, vector and doubly charged scalar) depending on the values of the couplings $a4$ and $a5$ ruling the dynamics of the symmetry breaking at the one-loop level. Through the study of these three unitarisations, we have been able to identify their strength and some of their weaknesses.

The strengths of the N/D method are that it preserves crossing symmetry and analyticity when utilised in its full capacity. The Padé approximation and K matrix approach, however, unitarise without necessarily accomodating analyticity. The K matrix approach doesn't give us any interesting data: it does not generate any resonances but, equally, it does not match the one-loop chiral perturbation theory calculations. The Padé approximation is a rigid method which gives us directly a first idea of the dynamics and possible induced resonances. The N/D seems to be the most rigorous and useful out of the three methods we have explored to date. It seems, naively, that the Padé and N/D methods agree to an extent. The Padé method is quite simple and so its conclusions straightforward. The N/D method, however, requires a more subtle interpretation. The comparison of these methods in order to reach a consistent view of the symmetry breaking certainly warrants further investigation. It would also give us an interesting report on the reliability of the unitarisation procedures used in the computation of scattering amplitudes.

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