NATURAL UNITS

When using natural units one sets each of the constants $\hbar$, k_B and c equal to unity. It is still possible to check the dimensions of a given equation, but one then has to understand that quantities such as those associated with velocity are dimensionless and that things like length and time have the same dimensions.

One can write every quantity in terms of powers of a single unit, e.g. GeV($= 10^9$ eV). The conversion factors to SI units are:

$$\begin{aligned}
\text{ENERGY} &: \quad 1\ \text{GeV} = 1.6 \times 10^{-10}\ \text{J} \\
\text{TEMPERATURE} &: \quad 1\ \text{GeV} = 1.16 \times 10^{13}\ \text{K} \\
\text{MASS} &: \quad 1\ \text{GeV} = 1.78 \times 10^{-27}\ \text{kg} \\
\text{LENGTH} &: \quad 1\ \text{GeV}^{-1} = 1.97 \times 10^{-16}\ \text{m} \\
\text{TIME} &: \quad 1\ \text{GeV}^{-1} = 6.58 \times 10^{-25}\ \text{sec}
\end{aligned}$$

Consider the present day critical density for a Hubble Constant of 100 $\text{km}\,\text{sec}^{-1}\text{Mpc}^{-1}$:

$$\rho_{\text{crit}} = 1.88 \times 10^{-29}\ \text{g cm}^{-3}\,. \tag{1}$$

In natural units

$$\text{g cm}^{-3} = \frac{1\text{GeV}}{1.78 \times 10^{-24}} \left(\frac{1\text{GeV}^{-1}}{1.97 \times 10^{-14}} \right)^{-3}\,, \tag{2}$$

and hence

$$\rho_{\text{crit}} = 8.07 \times 10^{-47}\text{GeV}^4\,. \tag{3}$$

Now consider the Hubble Constant, again taking 100 $\text{km}\,\text{sec}^{-1}\,\text{Mpc}^{-1}$. One can think of H_0^{-1} as either a length or a time (known as the Hubble radius and time respectively).

$$100\ \text{km}\,\text{sec}^{-1} = \frac{1}{3000}\,, \tag{4}$$

in natural units and hence (putting 1 MPc $= 3.1 \times 10^{22}$ m)

$$H_0^{-1} = 3000\ \text{Mpc} = 9.3 \times 10^{25} \left(\frac{6.58 \times 10^{-25}}{1.97 \times 10^{-16}} \right) \text{sec} = 3 \times 10^{17}\ \text{sec}\,. \tag{5}$$