First we solve for the scale factor in a matter-radiation Universe:

$a := rhs\left(dsolve\left(\left\{diff(a(t), t) = H\cdot\sqrt{\Omega_r + \Omega_m\cdot a(t)}, a(0) = 0\right\}, a(t)\right)\right)$

$$\frac{1}{4}\, t^2\, \Omega_m H^2 + t H \sqrt{\Omega_r} \quad (1)$$

Now fix the relevant cosmological parameters (conformal time t is in units of the inverse of the present day Hubble constant). Note I have increased the amount of radiation in order to delay the time of matter-radiation equality, which helps demonstrate the point that matter does not grow much during the radiation era once it has entered the horizon.

$\Omega_m := 1; \Omega_r := 0.1; H := 1 :$

$$1$$

$$0.1 \quad (2)$$

In these units, the present conformal time is

$tnow := fsolve(a = 1, t, 0..100)$

$$1.465162164 \quad (3)$$

and the conformal time of matter-radiation equality:

$teq := fsolve\left(\Omega_r = \Omega_m\cdot a, t, 0..100\right)$

$$0.2619716590 \quad (4)$$

So modes with inverse wavenumber smaller than

$evalf\left(\frac{2\cdot\pi}{teq}\right)^{-1}$

$$0.04169408447 \quad (5)$$

cross the horizon before matter-radiation equality.

Now fix the wavenumber (in units of the present day Hubble constant):

$k := 5000$

$$5000 \quad (6)$$

The perturbation equation we wish to solve simultaneously in terms of the matter (m) and radiation (r) perturbations are

$eq1 := \left(diff(m(t), t, t) + \frac{diff(a, t)}{a}\cdot diff(m(t), t) - \frac{3}{2}H^2\cdot\left(\frac{\Omega_m\cdot m(t)}{a} + \frac{2\cdot\Omega_r\cdot r(t)}{a^2}\right) = 0\right) :$

$eq2 := \left(diff(r(t), t, t) + \frac{k^2}{3}\cdot r(t) - \frac{4}{3} diff(m(t), t, t) = 0\right) :$

Set the adiabatic initial conditions:

$small := 10^{-6} :$

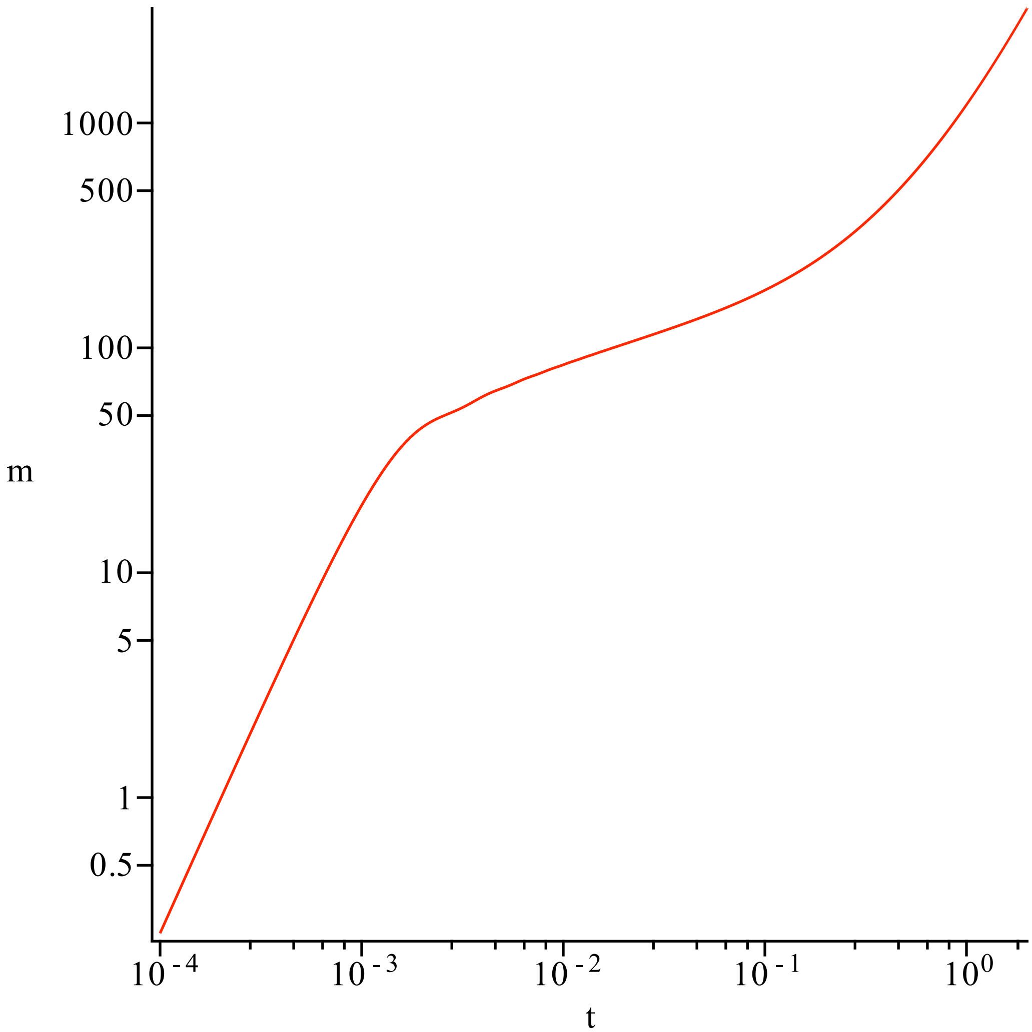
$inits := \left(m(small) = (k\cdot small)^2, r(small) = \frac{4}{3}\cdot(k\cdot small)^2, \mathrm{D}(m)(small) = 2\cdot k^2\cdot small, \mathrm{D}(r)(small) = \frac{8}{3}\cdot k^2\cdot small\right) :$

$soln := dsolve(\{eq1, eq2, inits\}, numeric, \{m(t), r(t)\}, maxfun = 0, range = 10^{-5}..2) :$

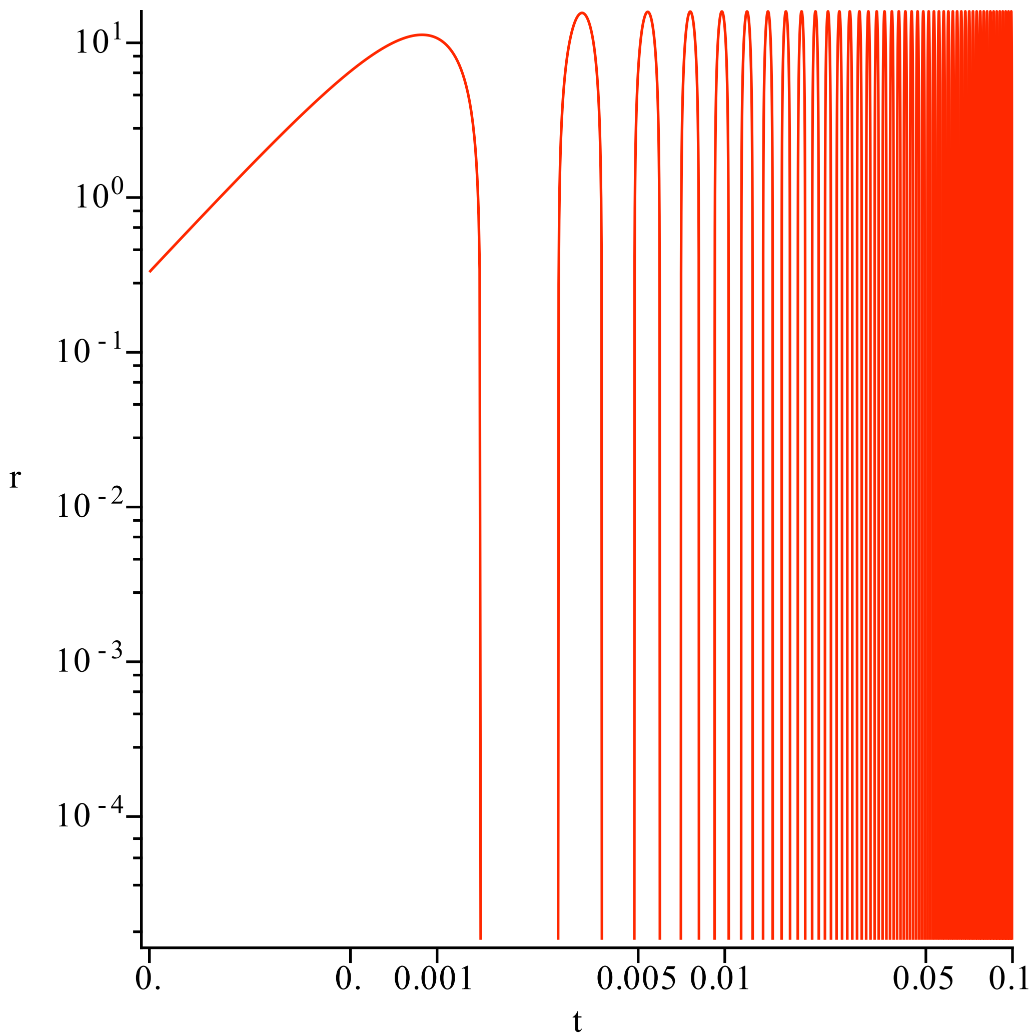
proc(*x_rkf45*) ... **end proc** **(7)**

with(*plots*) :

odeplot(*soln*, [*t*, *m*(*t*)], 10^{-4} ..2, *axis* = [*mode* = log], *refine* = 1)



odeplot(*soln*, [*t*, *r*(*t*)], 10^{-4} ..10^{-1}, *axis* = [*mode* = log], *refine* = 1)



We can clearly see that the mode enters the horizon around $t = 10^{-3}$, which is to be compared to the expected value of $\frac{2\,\pi}{k} = 1.3 \cdot 10^{-3}$. The slow rise in the radiation era is the Mezaros effect and the t^2 rise starts up again at the time of matter-radiation equality ($t = 0.26$).