

First define the pi\* energy band:

```
In[2]:= eplus[kx_, ky_] :=  
  Sqrt[1 + 4 * Cos[3 * kx / 2] * Cos[Sqrt[3] / 2 * ky] + 4 * Cos[ky * Sqrt[3] / 2]^2]
```

And then the 1st Brillouin zone and the K points:

```
In[3]:= K1 = 2 * Pi / 3 * {1, 1 / Sqrt[3], 0}
```

```
Out[3]= { 2 π / 3, 2 π / (3 √ 3), 0 }
```

```
In[4]:= K2 = 2 * Pi / 3 * {1, -1 / Sqrt[3], 0}
```

```
Out[4]= { 2 π / 3, - 2 π / (3 √ 3), 0 }
```

Check that the K points really are so:

```
In[5]:= eplus[K1[[1]], K1[[2]]]
```

```
Out[5]= 0
```

```
In[6]:= bz1 = {0, 4 * Pi / 3 / Sqrt[3], 0}
```

```
Out[6]= { 0, 4 π / (3 √ 3), 0 }
```

```
In[7]:= N[eplus[bz1[[1]], bz1[[2]]]]
```

```
Out[7]= 0.
```

```
In[8]:= bz2 = -K1
```

```
Out[8]= { - 2 π / 3, - 2 π / (3 √ 3), 0 }
```

```
In[9]:= bz3 = -K2
```

```
Out[9]= { - 2 π / 3, 2 π / (3 √ 3), 0 }
```

```
In[10]:= bz4 = -bz1
```

```
Out[10]= { 0, - 4 π / (3 √ 3), 0 }
```

```
In[11]:= size = 0.02
```

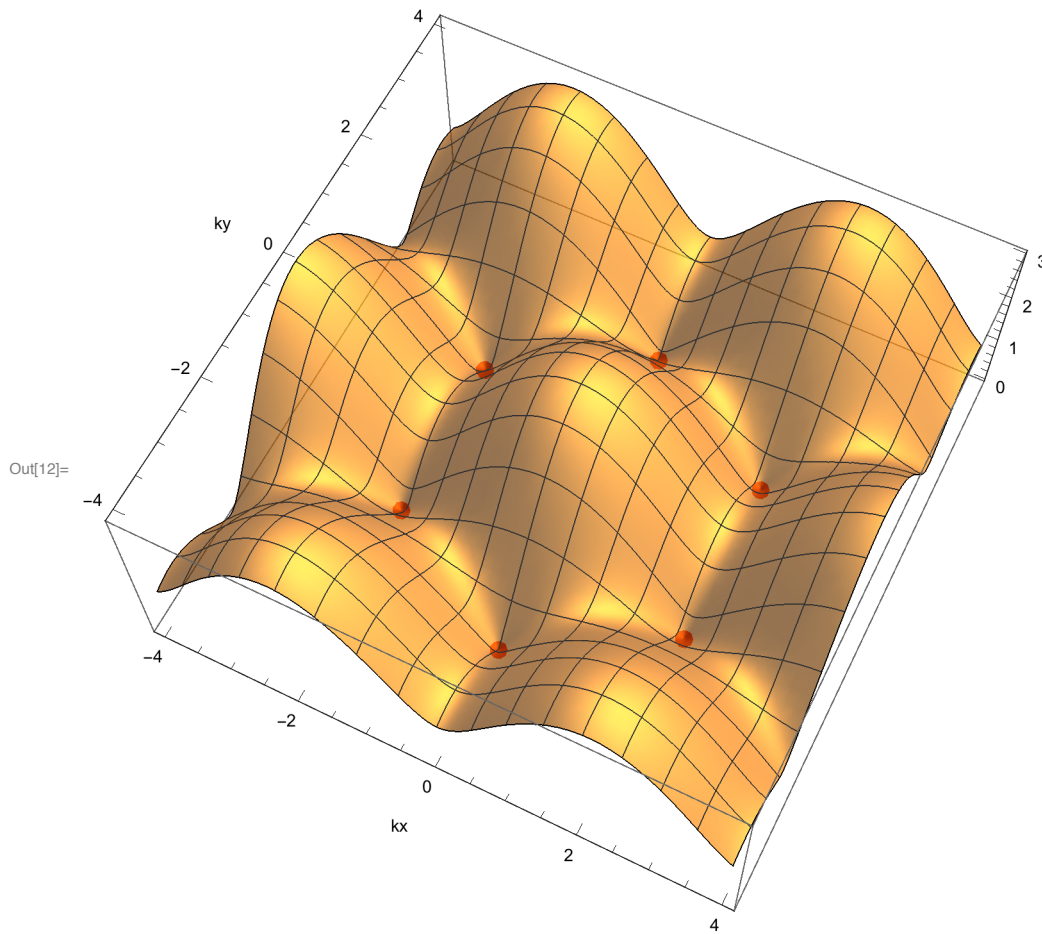
```
Out[11]= 0.02
```

The dots indicate the position of the corners of the 1st Brillouin zone.

```

In[12]:= Show[Plot3D[eplus[kx, ky], {kx, -4, 4}, {ky, -4, 4},
  PlotStyle -> {Opacity[0.7]}, PlotPoints -> 60, AxesLabel -> {kx, ky},
  Graphics3D[{Red, PointSize[size], Point[{K1, K2, bz1, bz2, bz3, bz4}]}}}

```



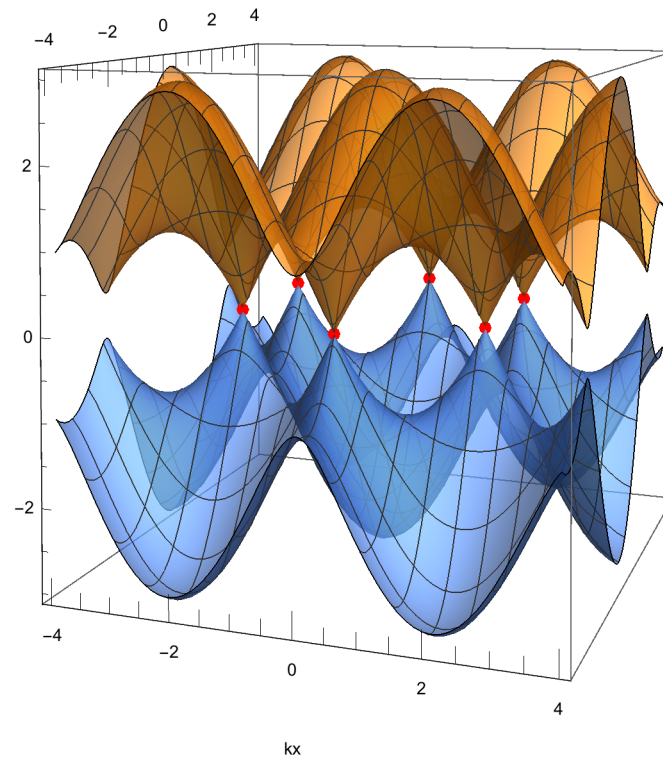
This is just the  $\pi^*$  band. Including the  $\pi$  band too:

```

In[13]:= Show[Plot3D[{eplus[kx, ky], -eplus[kx, ky]},
  {kx, -4, 4}, {ky, -4, 4}, PlotStyle -> {Opacity[0.7]},
  PlotPoints -> 60, AxesLabel -> {kx, ky}, AspectRatio -> 1],
  Graphics3D[{Red, PointSize[size], Point[{K1, K2, bz1, bz2, bz3, bz4}]}]]

```

Out[13]=



Now plot in the vicinity of a Dirac point to see the Dirac cones:

```

In[14]:= dk = 0.1

```

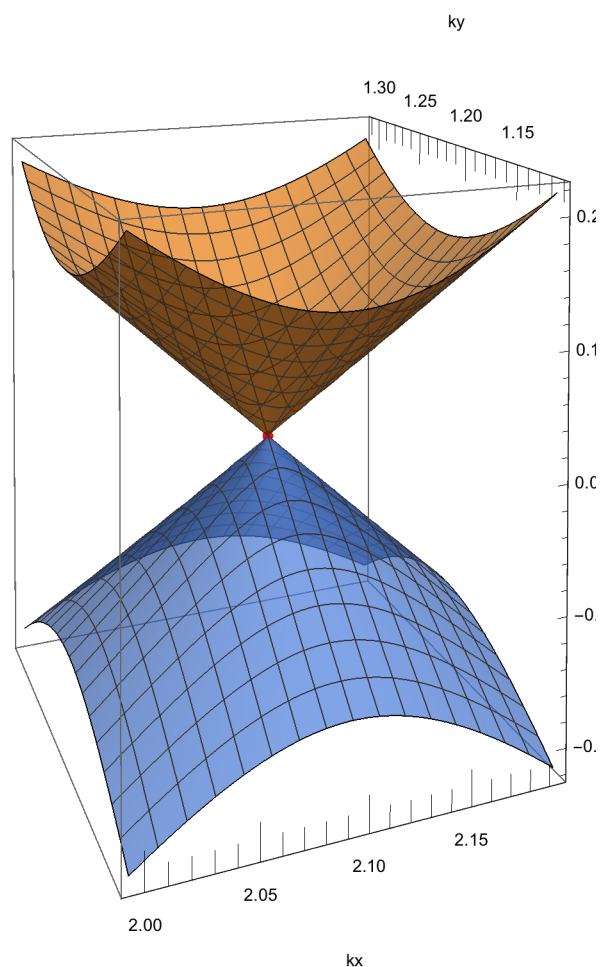
Out[14]= 0.1

```

In[15]:= Show[Plot3D[{eplus[kx, ky], -eplus[kx, ky]}, {kx, K1[[1]] - dk, K1[[1]] + dk},
  {ky, K1[[2]] - dk, K1[[2]] + dk}, PlotStyle -> {Opacity[0.7]},
  PlotPoints -> 60, AxesLabel -> {kx, ky}, AspectRatio -> 1.4],
  Graphics3D[{Red, PointSize[size], Point[K1]}]]

```

Out[15]=



Expanding about a Dirac point:

```

In[16]:= Series[eplus[2 * Pi / 3, 2 * Pi / 3 / Sqrt[3] + q], {q, 0, 1}]

```

Out[16]=  $\frac{3q}{2} + O[q]^2$

```

In[17]:= Series[eplus[2 * Pi / 3 + q, 2 * Pi / 3 / Sqrt[3]], {q, 0, 1}]

```

Out[17]=  $\frac{3q}{2} + O[q]^2$