EXAM QUESTION JANUARY 2011

1. Potential and a spherical shell

A conducting sphere of radius a, at a potential V_0, is surrounded by a thin concentric shell of charge. The radius of the shell is b and the surface charge density is:

$$\sigma(\theta) = k\cos\theta$$

where k is a constant and θ refers to spherical coordinates.

a) What are the boundary conditions on the scalar potential V and electric field $\vec{E}$ at $r = b$? [3 marks]

b) What is the electric field within the conductor ($r < a$)? [2 marks]

c) Find the electric potential inside ($a < r < b$) and outside ($r > b$) the surrounding shell by considering axially symmetric solutions to the Laplace equation, $\nabla^2 V = 0$. You should consider solutions of the form:

$$V(r,\theta) = \sum_{n=0}^{\infty}\left(A_n r^n + \frac{B_n}{r^{n+1}}\right)P_n(\cos\theta)$$

and you may find the following Legendre functions helpful in your analysis: $P_0(\cos\theta) = 1, P_1(\cos\theta) = \cos\theta$. Ensure that you clearly indicate all boundary conditions that are applied in your answer. [14 marks]

d) Find the surface charge $\sigma_i(\theta)$ on the conductor ($r=a$). [4 marks]

e) What is the total charge on the conductor ($r=a$)? [2 marks]

P.T.O.