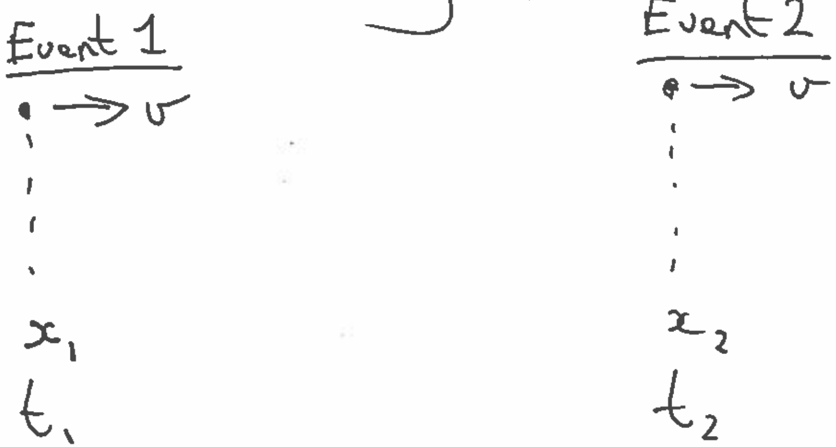
2024

Special Lecture — The Appearance of objects moving at close to the speed of light

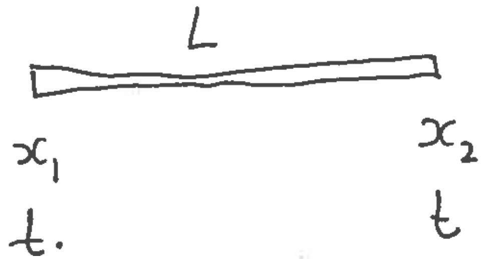
Contrast "appearance" with "measurements"

1) Speed of a moving particle (point)



$$v = \frac{x_2 - x_1}{t_2 - t_1}$$

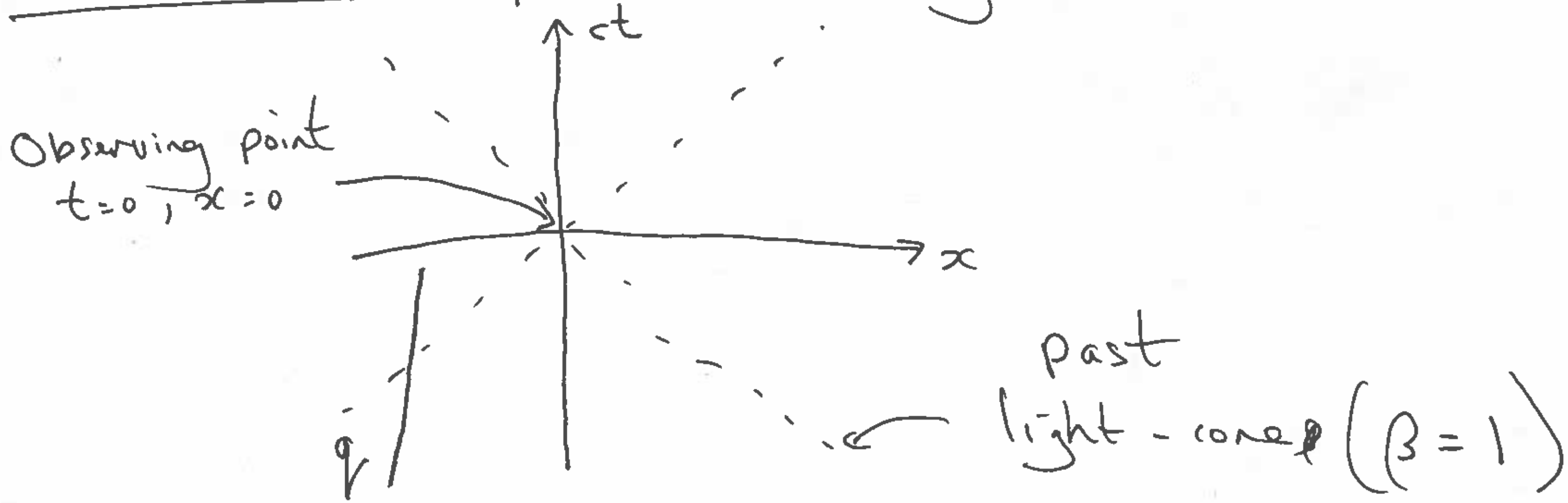
2) Length of moving rod



$$= \quad L = x_2 - x_1$$

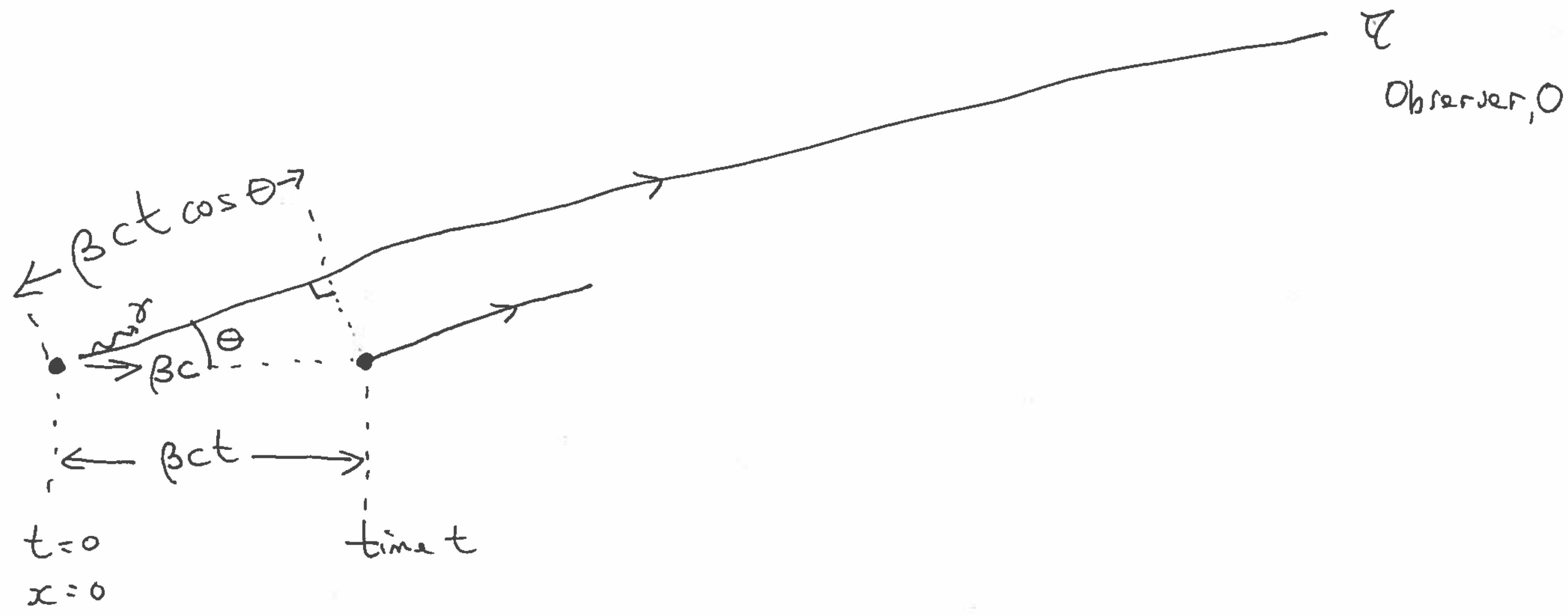
B) The appearance of objects : depends on the simultaneous arrival of light signals at a single observer

Interlude Space-time diagrams



world-line of a particle travelling at $\beta < 1$

Note: the world-line of q can cross the past light cone of observer at one point only.



Time difference for arrival at O of the two light signals

$$\Delta t = t - \beta t \cos\theta = t(1 - \beta\cos\theta)$$

Apparent speed of particle

$$v_{app} = \frac{\beta c t}{\Delta t} = \frac{\beta c}{1 - \beta\cos\theta} > \beta c$$

If $(1 - \beta \cos\theta) < \beta$ $\quad v_{app} > c$!

If particle is moving away then:

$$v_{app} = \frac{\beta c}{1 + \beta \cos\theta} < \beta c$$

In general:

$$v_{app} = \frac{\beta c}{\left[1 - \underline{\beta} \cdot \underline{\hat{R}}\right]_{ret}}$$

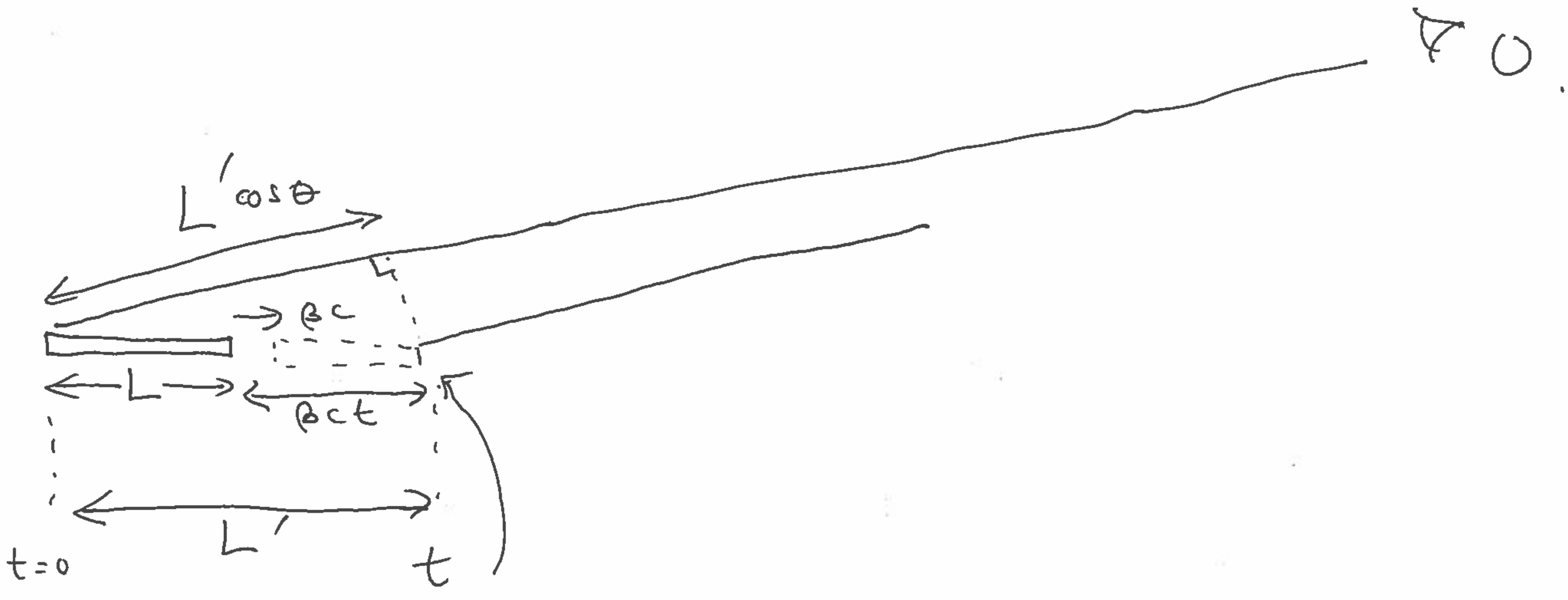
where $[\underline{\hat{R}}]_{ret}$ is the unit vector pointing from the position of particle to observer at the retarded time

What is the apparent length of a moving rod?

actual length L actual speed βc

↑

Actual length as measured in frame in which it is moving



$$L' = L + \beta c t \quad \text{(B)}$$ when the two light signals arrive simultaneously at O.

$$\therefore \quad t = \frac{L' \cos\theta}{c} \qquad \text{(A)}$$

Combine (A) and (B)

$$L' = \frac{L}{1 - \beta\cos\theta} > L$$

← same factor!

For the animation shown in the lecture and more interesting examples see:

https://www.spacetimetravel.org/bewegung/