Electrodynamics (PHYS30441)

Lecture 0: Introductory Comments

I hope to make this course a systematic and self-contained summary of Electrodynamics and Special Relativity starting from "first principles"

- Consistent notation to be used throughout the course
- You already have a good grounding in the ideas and mathematical tools so initially we can go quite quickly
- By taking this unified approach to Electromagnetism and Special Relativity I hope you will gain some interesting insights

Revision: Pre-recorded "Mini-Lectures" and Examples Class

- I have provided a set of pre-recorded "mini-lectures" that I'll be encouraging you to follow over the first three weeks of the course to help you revise the most relevant subjects in Electromagnetism and Special Relativity.
- The first official Examples Class (in week 3) is also devoted to "revision" problems.

Other Revision

You may find it useful to revise the notes from your 2nd year Electromagnetism course (PHYS20141)

– but you can leave out the sections on the fields in matter, e.g., the auxiliary fields ($\boldsymbol{D}, \boldsymbol{H}$), dielectrics, etc.

Useful also to revise 1st year Special Relativity from "Quantum Physics and Relativity" course (PHYS10121).

- I do not assume you've done the 1st year Advanced Dynamics course (PHYS10672), although those of you who have done this course might want to revise the section on Special Relativity.

We shall be dealing mainly with Electrodynamics in the vacuum

- A few problems will involve perfect conductors and perfect insulators

Much of this course concerns the connections between Special Relativity and Electromagnetism

- In the middle section of the course we shall re-write the laws of electromagnetism in the mathematical language of Special Relativity
- Historically, Maxwell's Equations for Potentials and Fields were consistent with Special Relativity before Special Relativity had been invented!

In this course we shall be dealing mainly with the Potentials (V, $\boldsymbol{A}$) and the Fields $(\boldsymbol{E}, \boldsymbol{B})$ rather than trying to calculate directly the forces among charged particles

A number of reasons for this

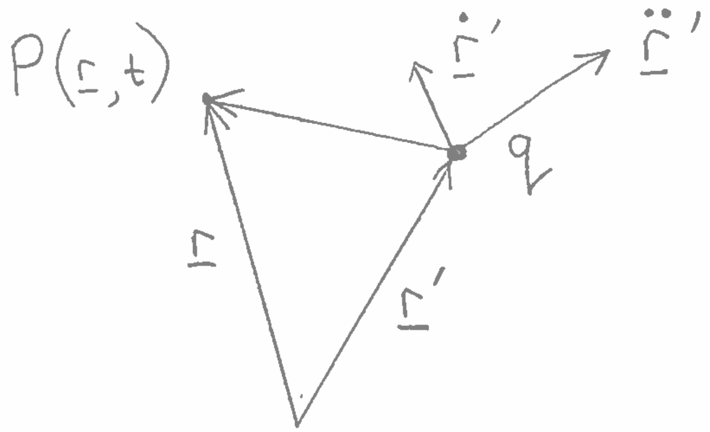
- Potentials and Fields lend themselves naturally to treatment in terms of Special Relativity.
  - Forces rather less so
- In many circumstances fields propagate in regions free of charges

If we do need to work out the force on charges we can use $\boldsymbol{F} = q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$ — Lorentz Force Law

In most cases we shall start with potentials $(V, \boldsymbol{A})$ from which we shall calculate (when necessary) the fields $(\boldsymbol{E}, \boldsymbol{B})$

- Often leads to more straightforward calculations
- Easier to understand intuitively
  - e.g., $(V, \boldsymbol{A})$ is of the form (scalar, 3-component vector)

- $*$ Does this look familiar???
- $*$ Has same form as $(t, \boldsymbol{r}) \rightarrow$ is it a 4-vector?



Another important consequence of Special Relativity is that the potentials and fields produced by moving charge particles propagate at the (finite) speed of light, c.

Consider Potentials and Fields at point $P(\boldsymbol{r}$,t) due to a moving point charge q

- Need to know $\boldsymbol{r'}, \dot{\boldsymbol{r}}', \ddot{\boldsymbol{r}}'$ at time $t_{\text{ret}} < t$.

t_{ret} is earlier than t due to time taken for signal to propagate distance $\boldsymbol{R} = |\boldsymbol{r} - \boldsymbol{r'}|$ from moving q to point P.

$$t_{\text{ret}} = t - \frac{R}{c}.$$

Note that $\boldsymbol{R}$ must be evaluated at the retarded time t_{ret}!

Already with a charge moving with constant velocity some interesting results can be obtained.

- In the final part of the course we shall derive expressions for the radiation from accelerating charges.

This relativistic treatment of "Classical" (i.e., non-Quantum) Electrodynamics provides a link into more advanced 4th year courses: Quantum Field Theory (PHYS40481), Gauge Theories (PHYS40682), etc.

There are two interactive sessions per week:

- Monday 15:00–16:00 (Simon lecture theatre A).
- Wednesday 9:00–10:00 (Stopford lecture theatre 6).

These interactive sessions will be used in a fairly flexible fashion.

- I shall use some of these sessions to give "live" lectures on some of the core material, to complement the set of pre-recorded mini-lectures.
- I'll organize some informal bonus "examples class"-style sessions, where we'll work through a problem together.
- I'm also happy to have some open "Q&A"-style sessions on the lecture material and exercises, if you would find these useful.
- I am happy for the balance among these various activities to be influenced to a degree by student opinions/requests; please let me know your views/suggestions!

During the (mini-)lectures I shall sometimes set small "exercises" for you, the students, to work out yourselves.

- You should regard these as an important element of the "core" material for the course, so please don't ignore them!
- I'll post "answers" to these in-line exercises at the end of the relevant week.

The web-site for this course is at:
https://users.hep.manchester.ac.uk/u/twyatt/electrodynamics/electrodynamics.html

- Within this there will be a separate page for each of the twelve weeks of the course, giving links to the interactive sessions, pre-recorded mini-lectures, recommendations for reading (for the current week and also for the next week), and miscellaneous other relevant material.

- N.B. I shall make explicitly clear in the page for each week of the course which mini-lectures are "duplicated" by a live lecture and, if appropriate, which mini-lectures need to be followed on-line, because the corresponding material is not covered in a live lecture.
- I provide also a single web-page summarising the entire set of pre-recorded mini-lectures from the course.

My scriblings on the Visualiser plus a one-page summary of each (mini-)lecture will be posted on the course web-site.

We'll be using Piazza for online Questions and Answers:

https://piazza.com/manchester.ac.uk/fall2024/phys30441

Electrodynamics problems are covered in the formal Examples Classes in weeks 3, 5, 8, 10, 12.

- Please, please make use of these!!!
- N.B. You can come along and ask me questions about the lectures, etc., as well as the formal Examples Class problems!

In order to study Electrodynamics we are inevitably going to be making quite heavy use of vector calculus

Vector calculus is my friend!

- A formula sheet and some exercises to help you revise vector calculus are provided. $\longrightarrow\longrightarrow\longrightarrow$
- In week 2 we'll have an informal interactive session to discuss some topics in vector calculus
  - including the use of index notation for 3D-vectors
    (to help prepare for our treatment of 4-vectors later in the course).

Blended learning approach, especially during the early "revision" phase of the course.

In this week's **Mini-Lectures 1, 2 and 3** we shall start with revision of Electrostatics

- Define some of the notation to be used throughout the course and practice using it in the context of electrostatics
- N.B. The revision material presented in Mini-Lectures 1, 2 and 3 will not be covered in a live lecture. Therefore you should follow revision Mini-Lectures 1, 2 and 3 online!

Suggestions for useful revision reading for this week

Vector Calculus

D.J.Griffiths, Introduction to Electrodynamics: Chapter 1.

Electrostatics

D.J.Griffiths, Introduction to Electrodynamics: Chapter 2.

Solutions to Laplace's equation

D.J.Griffiths, Introduction to Electrodynamics: Chapter 3.

N.B. Suggestions for further reading will be given on the web-page for each week of the course.

N.B. The book by Griffiths also has many problems that would form useful revision exercises.

Revision is necessary, but can be a bit dull!

Start our discussion of the fact that the potentials and fields produced by moving charge particles propagate at the speed of light, which is finite.

This fact is at the root of many of the aspects in which this course in Electrodynamics deviates from and extends your previous courses in Electromagnetism.

As an informal introduction to some of the weird and wonderful consequences of the finite speed of light we shall consider:
"The appearance of objects moving at close to the speed of light".

A few preliminaries on index notation for vectors in cartesian coordinates

We shall write the "i'th" component of a vector $\underline{u}$ in 3-dimensions as:

$$u_i = [\underline{u}]_i \, , \quad \text{where} \quad i = 1, 2, 3$$

and, in particular, $x_i = [\underline{r}]_i$

Because the three coordinates are orthogonal we have

$$\frac{\partial x_i}{\partial x_j} = \delta_{ij} \begin{cases} = 1 & \text{if } i = j \\ = 0 & \text{if } i \neq j \end{cases}$$

If an index is repeated in an expression it is summed over (unless explicitly stated to the contrary) so that, we can write:

$$\underline{u} = u_i \hat{\underline{x}}_i$$

e.g.

$$\nabla \cdot \underline{u} = \frac{\partial u_j}{\partial x_j}$$

$$\underline{u} \times \underline{v} = \varepsilon_{ijk} \hat{\underline{x}}_i u_j v_k$$

where

$$1 = \varepsilon_{123} = \underbrace{\varepsilon_{312} = \varepsilon_{231}}_{\text{cyclic permutations}} = -\varepsilon_{321} \text{ and other anti-cyclic permutations.}$$

$\therefore \varepsilon_{ijk} = 0$ if any pair of indices are equal.