Lecture 10) Special Relativity and 4-vectors

Postulates The laws of physics (results of experiments) are the same in all inertial frames of reference

including measuring the speed of light in vacuum.

Lorentz Transformation

Transform from coordinates in inertial frame, S, to another, S', moving with respect to S with velocity $\underline{v} = \beta c \hat{\underline{x}}$

$$[ct'] = \gamma([ct] - \beta x)$$

$$x' = \gamma(x - \beta[ct])$$

$$y' = y$$

$$z' = z$$

4-vectors

- Have structure (scalar, spatial vector)
- Components transform according to Lorentz Transformation

Examples

$$\underset{\sim}{x} = (ct, \underline{r}) = (ct, x, y, z) = (ct, x^i) = (x^0, x^1, x^2, x^3) = x^\mu$$

[where $i = x, y, z$
or $i = 1, 2, 3$]

[where $\mu = 1,2,3,4$]

$$\underset{\sim}{x}(1) - \underset{\sim}{x}(2) = \Delta \underset{\sim}{x} = (c\Delta t, \Delta \underline{r})$$

Scalar Product of Two 4-vectors

$$\underset{\sim}{a} \cdot \underset{\sim}{b} \equiv a^0 b^0 - \underline{a} \cdot \underline{b} = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

has same value in all inertial frames $\underset{\sim}{a}' \cdot \underset{\sim}{b}' = \underset{\sim}{a} \cdot \underset{\sim}{b}$

or "is invariant under a L.T." or "is a Lorentz invariant"

or "is a Lorentz scalar"