# Summary of Special Relativity in Minkowski Notation (Lecture 11)

## Contravariant (upper index)

$$x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$$

## Covariant (lower index)

$$x_\mu = (x_1, x_2, x_3, x_4) = (ct, -x, -y, -z)$$

## Lorentz Transformation

$$x'^0 = \gamma(x^0 - \beta x^1)$$
$$x'^1 = \gamma(-\beta x^0 + x^1)$$
$$x'^2 = x^2$$
$$x'^3 = x^3$$

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\Lambda^\mu{}_\nu = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x'_0 = \gamma(x_0 + \beta x_1)$$
$$x'_1 = \gamma(\beta x_0 + x_1)$$
$$x'_2 = x_2$$
$$x'_3 = x_3$$

$$x'_\mu = (\Lambda^{-1})^\nu{}_\mu x_\nu$$

$$(\Lambda^{-1})^\nu{}_\mu = \begin{bmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By considering the Lorentz transformations in both directions $x \to x'$ and $x' \to x$, for contra- and co-variant $\underset{\sim}{x}$ we can show that:

$$\left(\Lambda^{-1}\right)^{\mu}{}_{\nu} = \frac{\partial x'_{\nu}}{\partial x_{\mu}} = \frac{\partial x^{\mu}}{\partial x'^{\nu}}$$

$$\Lambda^{\mu}{}_{\nu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} = \frac{\partial x_{\nu}}{\partial x'_{\mu}}$$

## Metric Tensor

$$g_{\mu\nu} = g^{\mu\nu} = \begin{bmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{bmatrix} \qquad g_{\mu\lambda} g^{\lambda\nu} = \delta_\mu{}^\nu = \mathbb{1}_{4\times 4}$$

$x^\mu = g^{\mu\nu} x_\nu$ "raises Lorentz index"

$x_\mu = g_{\mu\nu} x^\nu$ "lowers Lorentz index"

## Scalar Product

$$a_\mu b^\mu = a^\mu b_\mu = a_\mu g^{\mu\nu} b_\nu = a^\mu g_{\mu\nu} b^\nu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

is a Lorentz scalar (invariant) $a'_\mu b'^\mu = a_\mu b^\mu$

## Magnitude

$x_\mu x^\mu = (ct)^2 - x^2 - y^2 - z^2$ is also Lorentz invariant.

## Summary of Lecture 11 The operator $\partial^\mu$

We define :

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \therefore \partial'_\mu = \frac{\partial}{\partial x'^\mu} = \left(\Lambda^{-1}\right)^\nu{}_\mu \frac{\partial}{\partial x^\nu}$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right) \therefore \partial'^\mu = \frac{\partial}{\partial x'_\mu} = \Lambda^\mu{}_\nu \frac{\partial}{\partial x_\nu}$$

That is :

Differential w.r.t. contravariant, $x^\mu$, transforms as covariant 4-vector

" " covariant, $x_\nu$, " " contravariant "