Lecture 12) 4-Vectors for Electrodynamics

4-velocity $u^\mu = \dfrac{dx^\mu}{d\tau} = (\gamma c, \gamma \underline{u})$; magnitude $u^\mu u_\mu = c^2$

is a 4-vector because dx^μ is a 4-vector and $d\tau$ is a Lorentz invariant

4-momentum $p^\mu = m u^\mu = m \dfrac{dx^\mu}{d\tau} = \left(\dfrac{E}{c}, \underline{p}\right)$, where $E = \gamma m c^2$, $\underline{p} = \gamma m \underline{u}$

is a 4-vector because u^μ is 4-vector and m is "rest mass"

magnitude $p^\mu p_\mu = (mc)^2 \implies E^2 - (pc)^2 = (mc^2)^2$

4-current $j^\mu = \rho_0 u^\mu = (\gamma \rho_0 c, \underbrace{\gamma \rho_0}_{} \underline{u}) = (\rho c, \rho \underline{u}) = (\rho c, \underline{j})$

where charge density: ρ_0 ↑ rest frame; ρ frame in which charge is moving

Continuity Equation for Charge

$\dfrac{1}{c}\dfrac{\partial}{\partial t}(c\rho) + \nabla \cdot \underline{j}$ may be written as $\partial_\mu j^\mu = 0$

A nice compact form, but more importantly guarantees "Lorentz Covariance"

$\partial'_\mu j'^\mu = 0$ Law takes the same form in all inertial frames.

Makes explicit the link between the scalar and spatial vector components

Space time interval between two points on the world line of a particle

$\Delta x_\mu \Delta x^\mu = c^2 \Delta \tau^2$ which is Lorentz invariant

The "proper time" interval $\Delta\tau$ is that measured by a clock at rest w.r.t. the particle.

In frames in which the particle is moving $\Delta t = \gamma \Delta \tau$ (time interval between points on the world line of the particle