

Putting this together

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{matrix} \mu \searrow \nu \rightarrow \\ \downarrow \end{matrix} \begin{bmatrix} 0 & -\frac{E_1}{c} & -\frac{E_2}{c} & -\frac{E_3}{c} \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{bmatrix}$$

The Electromagnetic Field Tensor

There are only six independent terms: $\underline{E}, \underline{B}$

Exercise : Verify explicitly each element of $F^{\mu\nu}$!

Reminder

$$\begin{aligned}
 \tilde{F}_2' &= F^{120} = \Lambda^2_\alpha \Lambda^0_\beta F^{\alpha\beta} \\
 &= \Lambda^2_2 \left(\Lambda^0_0 F^{20} + \Lambda^0_1 F^{21} \right) \\
 &= \gamma \left(\frac{E_2}{c} - \beta B_3 \right)
 \end{aligned}$$

Exercise : work out explicitly the other transformations.