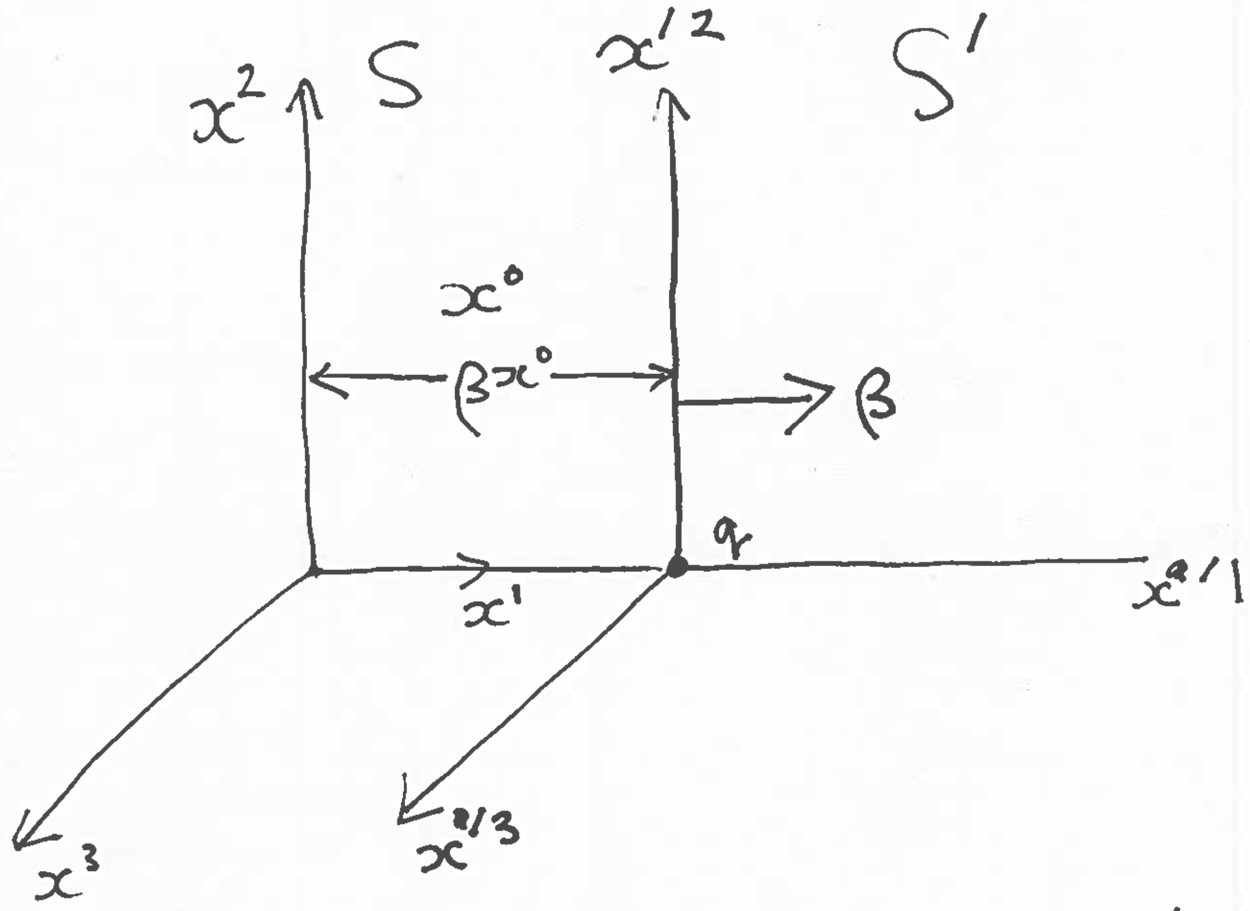
Lecture 14) Potentials for a point charged particle moving q with constant velocity β along x^1 axis



charge q at rest at origin of S'

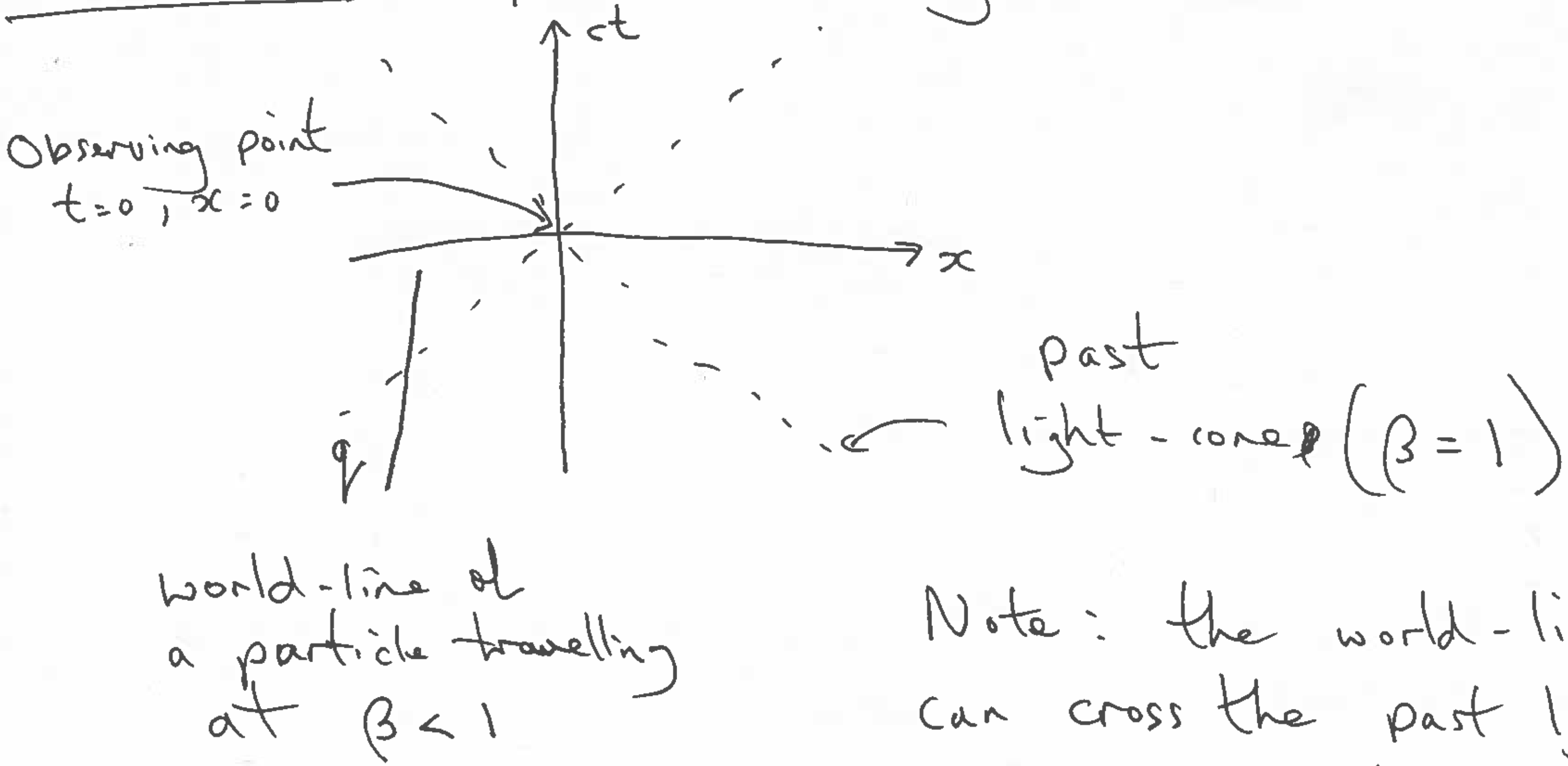
We are going to evaluate potential A^μ in frame S at "time" x^0

N.B. Need to transform A'^μ but also $x'^1 = \gamma[x^1 - \beta x_0]$

Reminder from Special Lecture in Week 1:

B) The appearance of objects: depends on the simultaneous arrival of light signals at a single observer

<u>Interlude</u> Space-time diagrams



Note: the world-line of q can cross the past light cone of observer at one point only.

In rest frame of q S':

$$A'^{0} = \frac{V'}{c} = \frac{q}{4\pi\varepsilon_0 c}\frac{1}{R'}, \quad \underline{A}' = 0$$

where $(R')^2 = (x'^1)^2 + (x'^2)^2 + (x'^3)^2$

$$= \left(\gamma\left[x^1 - \beta x^0\right]\right)^2 + (x^2)^2 + (x^3)^2 \qquad (14.1)$$

Potentials in frame S

L.T. $\frac{V}{c} = A^0 = \gamma\left(A'^0 + \beta A'^1\right) = \gamma A'^0$

$$= \frac{q\gamma}{4\pi\varepsilon_0 c R'} = \frac{q}{4\pi\varepsilon_0 c}\gamma \frac{1}{\left[\left(\gamma\left[x^1-\beta x^0\right]\right)^2 + \left(x^2\right)^2 + \left(x^3\right)^2\right]^{1/2}} \qquad (14.2)$$

$$A^1 = \gamma\left(\beta A'^0 + A'^1\right) = \gamma\beta A'^0 = \beta A^0 = \frac{q\gamma\beta}{4\pi\varepsilon_0 c R'} \qquad (14.3)$$

$$A^2 = A^3 = 0$$

Exercise: Verify that $A'^{\mu} A'_{\mu} = A^{\mu} A_{\mu}$

What do these potentials "look like" in frame S?

Express A^μ in terms of R

Special case 1

Position is on x^1 axis

$$x^2 = x^3 = 0$$

Substitute into (14.2)

$$A^0 = \frac{V}{c}\,\frac{q}{4\pi\varepsilon_0 c}\,\cancel{\gamma}\,\frac{1}{\cancel{\gamma}\,|x^1 - \beta x_0|}$$

That is, the same potential as for a stationary charge q at ~~a~~ the "current" position $x^1 = \beta x^0$, $x^2 = x^3 = 0$!

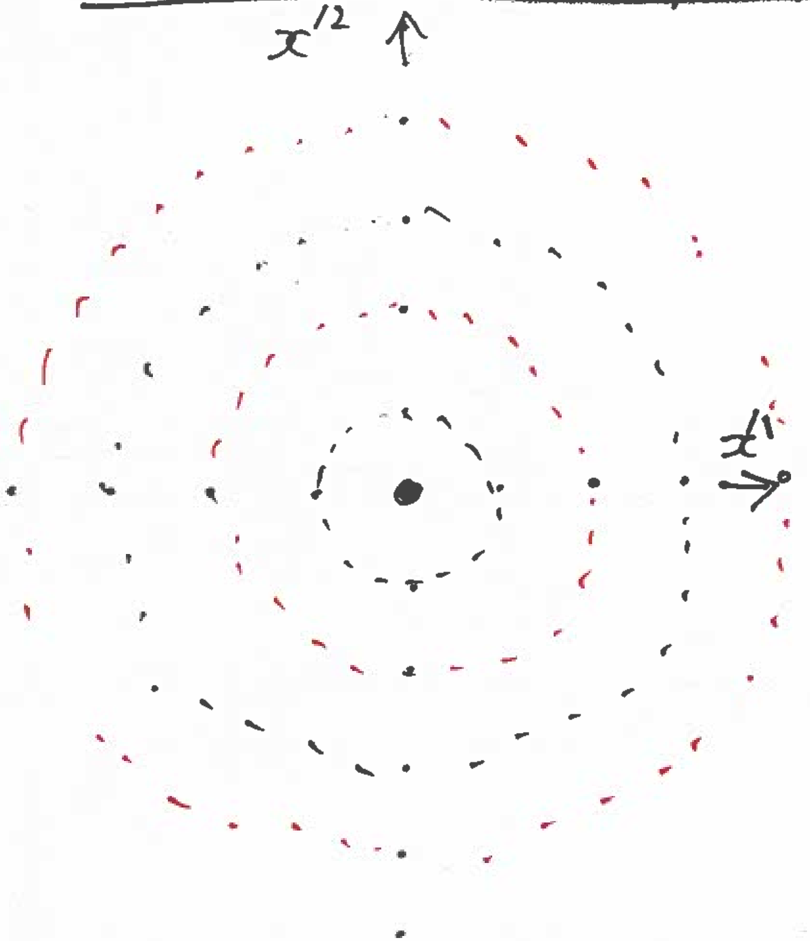
Special case 2)

Position transverse to x^1 axis at the "current" position $x^1 = \beta x^0$ of q.

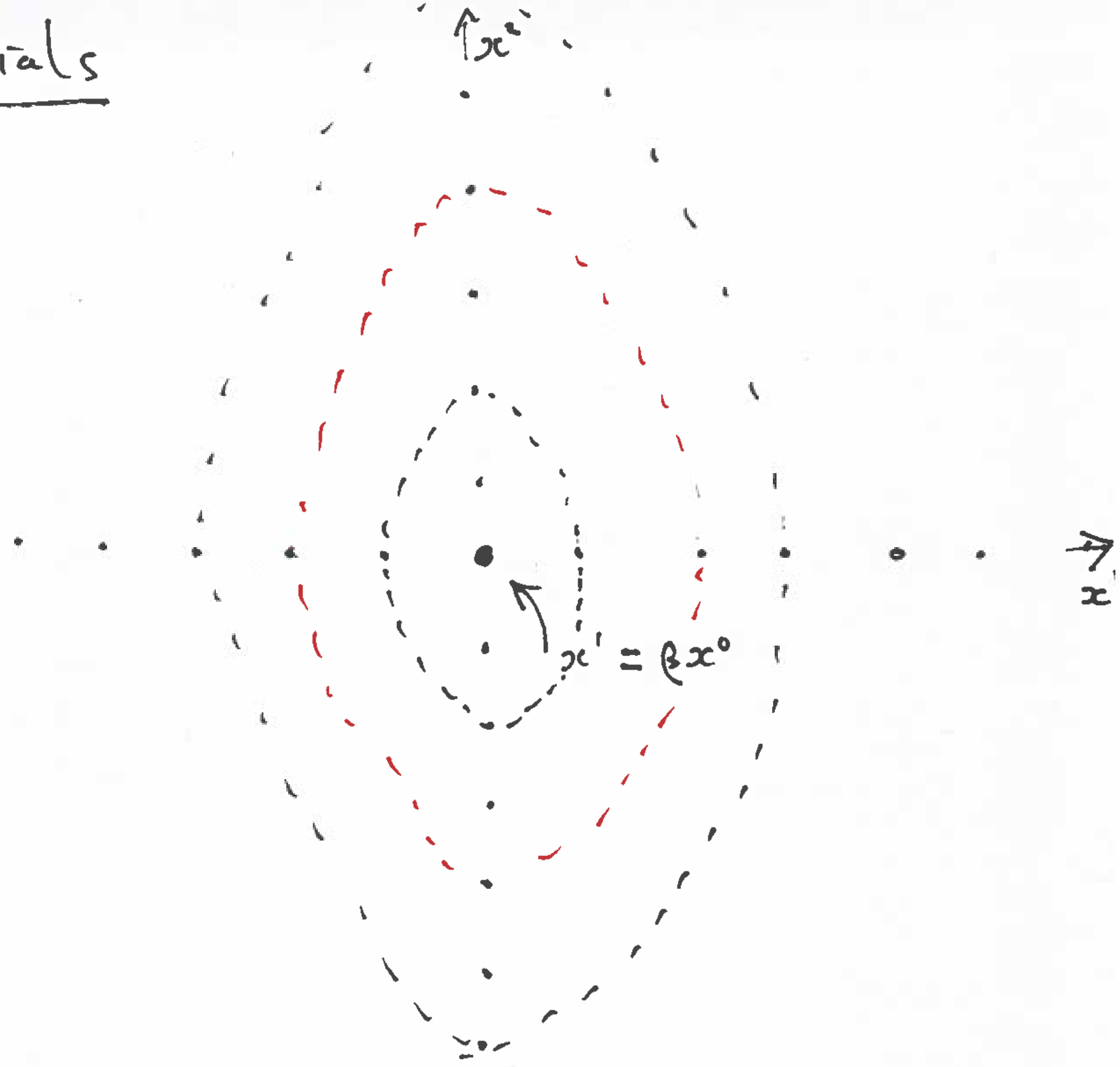
$$A^0 = \frac{V}{c} = \gamma \left[\frac{q}{4\pi\varepsilon_0 c} \frac{1}{\left[(x^2)^2 + (x^3)^2 \right]^{1/2}} \right]$$

That is, γ times the potential for a stationary charge at the "current" position $x^1 = \beta x^0$, $x^2 = x^3 = 0$

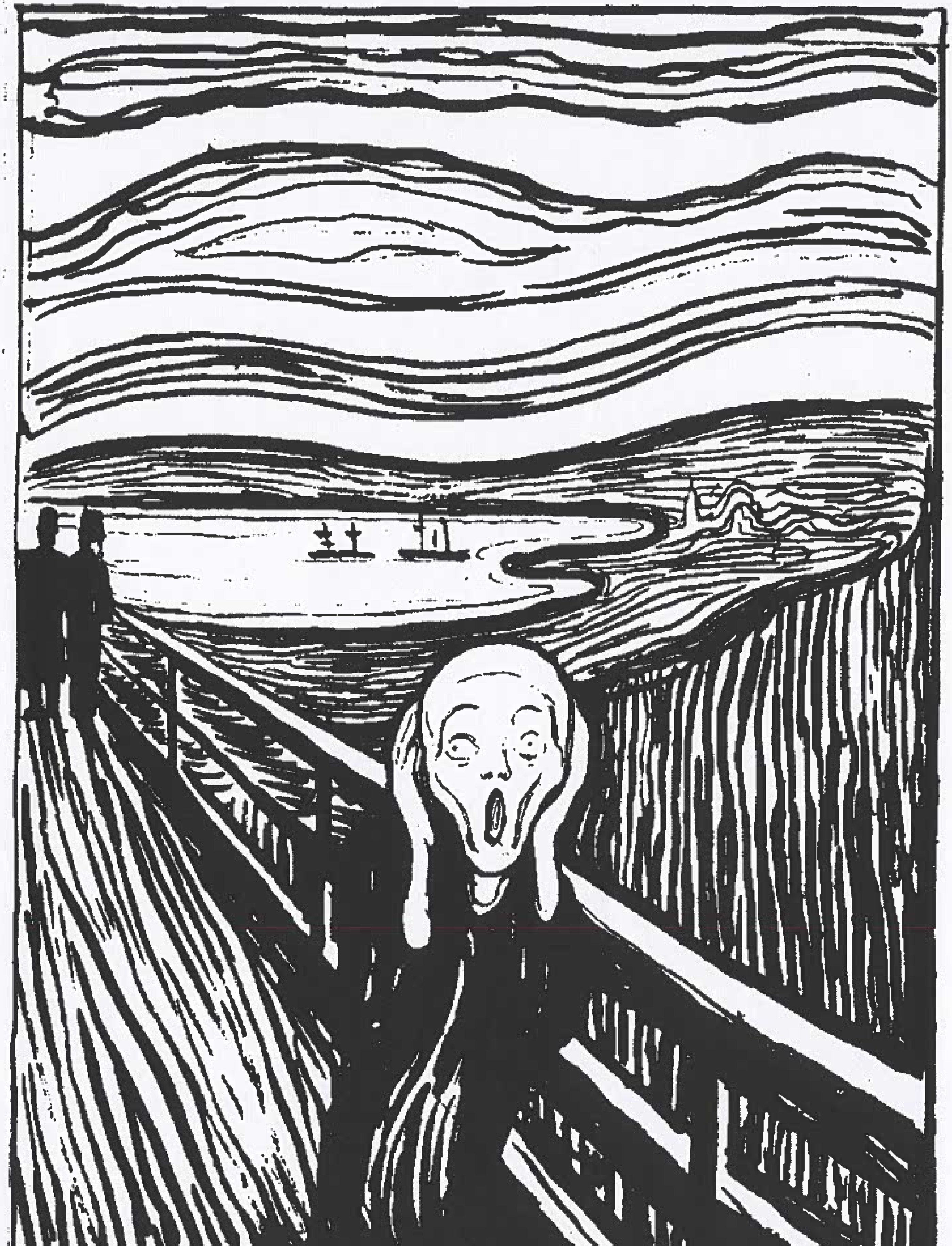
Sketches of equipotentials



Stationary charge



Moving q with $\gamma = 2$



Suprisingly, these equipotentials are centred at the "current position" $x^1 = \beta x^0$ of q

actual position of q at time x^0

and not the "retarded position"!

from where the electromagnetic signal actually originated.