Lecture 14) Potentials for a point charge moving with constant velocity

$P(\underline{r}, t)$ at which we determine the potentials is connected by a light signal with only on point on the past world line of the point charge

Point charge q at rest at origin in frame S'

We need to transform A'^{μ} but also $x'^{1} = \gamma[x^1 - \beta x^0]$

In rest frame S': $A'^{0} = \frac{V'}{c} = \frac{q}{4\pi\varepsilon_0 c} \frac{1}{R'}$, $\underline{A}' = 0$

where $(R')^2 = (x'^{1})^2 + (x'^{2})^2 + (x'^{3})^2 = (\gamma[x^1 - \beta x^0])^2 + (x^2)^2 + (x^3)^2$

In frame S charge moves with velocity $\underline{\beta} = \beta \hat{\underline{x}}^1$:

$$A^0 = \frac{q}{4\pi\varepsilon_0 c} \gamma \frac{1}{[(\gamma[x^1 - \beta x^0])^2 + (x^2)^2 + (x^3)^2]^{1/2}} = \gamma A'^{0}$$

$$A^1 = \beta A^0 = \gamma\beta A'^{0} , \qquad A^2 = A^3 = 0$$

Note: Equipotentials are centred at the current position, $x^1 = \beta x^0$, of the charge and not at the position at the retarded time!