Lecture 15) $\underline{E}$ and $\underline{B}$ fields for a moving point charge q.

In rest frame S'

$$\frac{\underline{E}'}{c} = \frac{q}{4\pi\varepsilon_0 c}\frac{\underline{R}'}{(R')^3}$$

$$\frac{E_1'}{c} = \frac{q}{4\pi\varepsilon_0 c}\frac{x'^1}{(R')^3} = \frac{q}{4\pi\varepsilon_0 c}\frac{\gamma[x^1 - \beta x^0]}{(R')^3} \qquad (15.1)$$

$$\left.\begin{aligned}\frac{E_2'}{c} &= \qquad = \frac{q}{4\pi\varepsilon_0 c}\frac{x^2}{(R')^3}\\ \frac{E_3'}{c} &= \qquad = \quad ?\end{aligned}\right\} \qquad (15.2)$$

Exercise: Fill in the blanks!

<u>Method 1</u> From expression for A^μ in frame S (Lecture 14) and rules for finding $F^{\mu\nu}$ from A^μ (Lecture 1

$$\frac{E_1}{c} = F^{10} = \partial^1 A^0 - \partial^0 A^1 = -\frac{\partial A^0}{\partial x^1} - \frac{\partial A^1}{\partial x^0}$$

$$= -\frac{q\gamma}{4\pi\varepsilon_0 c}\left[\frac{\left(-\frac{1}{2}\right)2\gamma^2[x^1-\beta x^0]}{(R')^3} + \frac{\beta\left(-\frac{1}{2}\right)2\gamma^2[x^1-\beta x^0](-\beta)}{(R')^3}\right]$$

$$= \frac{q\gamma}{4\pi\varepsilon_0 c}\frac{[x^1-\beta x^0]}{(R')^3}\,\underbrace{\gamma^2(1-\beta^2)}_{=1} \qquad (15.3)$$

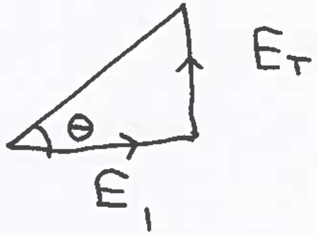
$$\frac{E_2}{c} = F^{20} = \partial^2 A^0 - \partial^0 A^2 = -\frac{\partial A^0}{\partial x^2} = \qquad = \frac{q\gamma}{4\pi\varepsilon_0 c}\frac{x^2}{(R')^3}$$

$$\frac{E_3}{c} = \qquad = \qquad = \qquad = \qquad = \quad x^3$$

Fill in the blanks!

(15.4

Magnitude of $\underline{E}$ in S



$$\left((x^2)^2+(x^3)^2\right)^{1/2} = (\ )$$

$$[x^1-\beta x^0] = [\ ]$$

$$|\underline{E}| = \left(E_1^2 + E_T^2\right)^{1/2}$$

$$= \frac{q\gamma}{4\pi\varepsilon_0}\left(\frac{[\ ]^2+(\ )^2}{\left\{\gamma^2[\ ]^2+(\ )^2\right\}^3}\right)^{1/2}$$

Note $\quad R^2 = [\ ]^2 + (\ )^2, \quad \sin\theta = \dfrac{(\ )}{R}, \quad \cos\theta = \dfrac{[\ ]}{R}$

Exercise: Show that we can write:

$$|\underline{E}| = \frac{q}{4\pi\varepsilon_0}\frac{1}{R^2}\frac{1}{\gamma^2}\left(\frac{1}{1-\beta^2\sin^2\theta}\right)^{3/2} \qquad (15.7)$$

B fields for moving q

$$B_i = -F^{jk} = -\left(\partial^j A^k - \partial^k A^j\right)$$

$$A^1 = \frac{q}{4\pi\varepsilon_0 c}\frac{\gamma\beta}{R'} , \quad A^2 = 0, \quad A^3 = 0$$

$$B_1 = 0$$

$$B_2 = -\partial^3 A^1 = \frac{\partial A^1}{\partial x^3} \qquad = -\frac{\beta E_3}{c}$$

$$B_3 = \qquad = \frac{\beta E_2}{c}$$

(15.8)

$$\therefore \underline{B} = \frac{1}{c}\left(\underline{\beta} \times \underline{E}\right) \qquad (15.9)$$

Fill in the blanks