Lecture 12 E and B Fields Produced by Moving Point Charge (Method 2)

Lorentz transformation is more complicated for tensors of rank 2:

$$F'^{\mu\nu} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} F^{\alpha\beta}$$

For each specific element of $F'^{\mu\nu}$, the sums over α, β yield only one or non-zero terms! Alteratively, writing in matrix form:

$$F' = \Lambda F \Lambda$$

it can be shown that:

$$\frac{E'_1}{c} = \frac{E_1}{c} \qquad B'_1 = B_1$$

$$\frac{E'_2}{c} = \gamma\left(\frac{E_2}{c} - \beta B_3\right) \qquad B'_2 = \gamma\left(B_2 + \beta \frac{E_3}{c}\right)$$

$$\frac{E'_3}{c} = \gamma\left(\frac{E_3}{c} + \beta B_2\right) \qquad B'_3 = \gamma\left(B_3 - \beta \frac{E_2}{c}\right)$$

Applying inverse transformations $S' \Rightarrow S$, $\beta \Rightarrow -\beta$ to the fields in S' yields $\underline{E}$ and $\underline{B}$ consistent with equations (14.3), (14.4) and (14.8).