Lecture 16) Maxwell's Equations, Interaction of Charged Particles with Fields

In Lorentz-covariant form:

Maxwell's Equations
$$\partial_\mu F^{\mu\nu} = \mu_0 j^\nu$$
$$\partial^\mu F^{\nu\lambda} + \partial^\lambda F^{\mu\nu} + \partial^\nu F^{\lambda\mu} = 0$$

Wave Equation
$$\partial_\mu F^{\mu\nu} = \partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = \partial_\mu \partial^\mu A^\nu - \partial^\nu(\partial_\mu A^\mu)$$
$$= \Box^2 A^\nu \quad \text{if } \partial_\mu A^\mu = 0 \quad \text{(Lorenz gauge)}$$

Interaction of charged particle with the fields

$$\frac{dp^\mu}{d\tau} = q F^{\mu\nu} u_\nu \quad \text{equivalent to} \quad \underset{\text{energy of particle}}{\frac{d\varepsilon}{dt}} = q\,\underline{u}\cdot\underline{E}\,;\ \frac{d\underline{p}}{dt} = q(\underline{E} + \underline{u}\times\underline{B})$$

Relativistic Kinematics

$\sum_i \underset{\sim}{p}_i = \sum_f \underset{\sim}{p}_f$, where i are the initial- and f the final-state particles.

A useful trick: Isolate 4-momentum of particle you are not interested in $\underset{\sim}{P}\cdot\underset{\sim}{P} = m^2c^2$ for this particle. (See Compton Scattering example)