Lecture 17 (part 1) Local Conservation Laws / Symmetries

"Action at a distance", "global" conservation laws, are not compatible with Special Relativity $\Longrightarrow$ Require "local" conservation laws/symmetries

E.g., Local conservation of charge : $\partial_\mu j^\mu = 0$ or $\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{j} = 0$

(Charge is conserved at each point in space-time)

E.g., Local gauge symmetry : $A^\mu \Rightarrow A^\mu - \partial^\mu \chi$

$\left\{ \begin{array}{l} \chi \text{ may be chosen separately at each point in space-time,} \\ \text{subject to the appropriate gauge condition, such as } \partial_\mu A^\mu = 0 \end{array} \right\}$

E.g., Local conservation of energy in electrodynamics

$$\underline{E} \cdot \underline{j} + \nabla \cdot \underline{S} + \frac{\partial u}{\partial t} = 0$$

where $\underline{S} = \frac{1}{\mu_0} \underline{E} \times \underline{B}$ (Poynting vector)

and $u = \frac{\varepsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$ (energy density in fields)

Lecture 17 (part 2) Retarded Potentials for a moving point charge

At retarded time $t' = t - \frac{R}{c}$ charge q is moving with velocity $\underline{v} = \underline{\beta} c$ and displacement of observation point $P(\underline{r}, t)$ relative to charge is $\underline{R}$.

"Effective charge" seen at $P(\underline{r}, t)$ is

$$\frac{q}{\left[1 - \underline{\beta} . \hat{\underline{R}}\right]_{ret}}$$

The "ret" reminds us that $\underline{\beta}$, $\hat{\underline{R}}$ are evaluated at retarded time.