Exercise for you: show that

$$\left[[\ ]^2 + (1-\beta^2)(\ )^2 \right] = \left[R_{ret}(1 - \beta\cos\theta) \right]^2$$

$$= \left[R_{ret}(1 - \underline{\beta} \cdot \hat{R}_{ret}) \right]^2 \qquad (19.6)$$

Exercise: show that $\nabla R = \hat{\underline{R}}$

Using V.I. (4)

$$-\nabla(\underline{\beta}\cdot\underline{R}) = \underbrace{-\underline{\beta}\times(\nabla\times\underline{R})}_{\to 0 \text{ as } \beta\to 0} - \underline{R}\times(\nabla\times\underline{\beta}) - \underbrace{(\underline{\beta}\cdot\nabla)\underline{R}}_{\to 0 \text{ as } \beta\to 0} - (\underline{R}\cdot\nabla)\underline{\beta} \quad (19.11)$$

$$\nabla\times\underline{\beta} = \varepsilon_{ijk}\frac{\partial}{\partial x^j}\beta^k = \varepsilon_{ijk}\frac{\partial t_{ret}}{\partial x^j}\frac{\partial\beta^k}{\partial t_{ret}} = \nabla t_{ret}\times\dot{\underline{\beta}} \quad (19.2)$$

where we have defined $\dot{\underline{\beta}} = \dfrac{\partial\underline{\beta}}{\partial t_{ret}}$

$$t_{ret} = t - \frac{R}{c} \underset{\beta\to 0}{\Longrightarrow} \nabla t_{ret} = -\frac{\nabla R}{c} = -\frac{\hat{R}}{c} \quad (19.13)$$