Some useful relations

$$(\;) = R_{ret} \sin\theta \qquad (19.2)$$

$$x^1 - \beta x^0_{ret} = R_{ret} \cos\theta \qquad (19.3)$$

From $\quad t_{ret} = t - \dfrac{R_{ret}}{c}$

$$x^0_{ret} + R_{ret} = x^0 \qquad (19.4)$$

$$\therefore \quad [\;] = x^1 - \beta\left(x^0_{ret} + R_{ret}\right) = R_{ret}\left(\cos\theta - \beta\right) \qquad (19.5)$$

(using 19.3)

Exercise : Show that

$$\left\{ [\;]^2 + (1-\beta^2)(\;)^2 \right\} = \left[R_{ret}\left(1 - \beta\cos\theta\right)\right]^2$$

$$= \left[R_{ret}\left(1 - \underline{\beta} \cdot \hat{\underline{R}}_{ret}\right)\right]^2 \qquad (19.6)$$

Using (19.2) and (19.5) we can express

$$\boxed{\alpha = \theta}$$

$$\{ [x^1 - \beta x^0]^2 + (1-\beta)^2 (\ )^2 \}$$

$$= R_{ret}^2 \left(\cos^2\alpha - 2\beta\cos\alpha + \beta^2 + (1-\beta^2)\sin^2\alpha \right)$$

$$= R_{ret}^2 \left(\cos^2\alpha + \sin^2\alpha - 2\beta\cos\alpha + \beta^2(1 - \sin^2\alpha) \right)$$

$$= R_{ret}^2 \left(1 - 2\beta\cos\alpha + \beta^2\cos^2\alpha \right) = \left[R_{ret}(1 - \beta\cos\alpha) \right]^2$$

$$= \left[R_{ret}\left(1 - \underline{\beta}\cdot\hat{\underline{R}}_{ret}\right) \right]^2 \qquad (19.6)$$

Part 2) Use vector calculus to find $\underline{E}$, $\underline{B}$ produced by an accelerating point charge

Consider the case $\beta \ll 1$ or $\beta \to 0$

Just for brevity I'll drop "ret" from R_{ret}

$$\underline{E} = -\nabla V - \frac{\partial \underline{A}}{\partial t} \qquad (19.9)$$

$$-\nabla V = -c\nabla A^0 = \frac{-q}{4\pi\varepsilon_0} \frac{-1}{[R - \underline{\beta}\cdot\underline{R}]^2} \left(\nabla R - \nabla(\underline{\beta}\cdot\underline{R})\right) \qquad (19.10)$$

Exercise for you: show that $\nabla R = \hat{\underline{R}}$

Remembering that here $\underline{R}$ stands for $\underline{R}_{ret} = \underline{r} - \underline{r}_{ret}$

$$R = \left[(x - x_{ret})^2 + (y - y_{ret})^2 + (z - z_{ret})^2\right]^{1/2}$$

$$\nabla R = \frac{1}{2}\frac{1}{R} 2\left[\hat{\underline{x}}(x - x_{ret}) + \hat{\underline{y}}(y - y_{ret}) + \hat{\underline{z}}(z - z_{ret})\right]$$

$$= \frac{\underline{R}}{R} = \hat{\underline{R}}$$

Alternatively, using index notation

$$\nabla R = \hat{\underline{x}}^i \frac{\partial}{\partial x^i}\left[(x^j - x^j_{ret})(x^j - x^j_{ret})\right]^{1/2}$$

$$= \hat{\underline{x}}^i \frac{1}{2}\frac{1}{R} 2\left[\delta_{ij}(x^j - x^j_{ret})\right] = \frac{\hat{\underline{x}}^i (x^i - x^i_{ret})}{R}$$

$$= \frac{\underline{R}}{R} = \hat{\underline{R}}$$