

Continuation from Lecture 19

Vector Calculus.

The first part of the lecture was devoted to the study of the properties of the vector field $\vec{F}$ and the scalar field ϕ .

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$$\vec{F} = \vec{F}(\vec{r}) = \frac{\vec{r}}{r^3} \quad \vec{r} = \frac{\vec{r}}{r}$$

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Use (19.12) & (19.3)

$$-\underline{R} \times (\nabla \times \underline{\beta}) = \frac{1}{c} \underline{R} \times (\hat{\underline{R}} \times \dot{\underline{\beta}}) \quad (19.14)$$

$$\begin{aligned} -(\underline{R} \cdot \nabla) \underline{\beta} &= -x^i \frac{\partial}{\partial x^i} \underline{\beta} = -x^i \frac{\partial t_{\text{ret}}}{\partial x^i} \frac{\partial \underline{\beta}}{\partial t_{\text{ret}}} \\ &= (\underline{R} \cdot \nabla t_{\text{ret}}) \dot{\underline{\beta}} = \frac{1}{c} (\underline{R} \cdot \hat{\underline{R}}) \dot{\underline{\beta}} = \frac{R}{c} \dot{\underline{\beta}} \quad (19.15) \\ &\quad \text{(using 19.13)} \end{aligned}$$

Substitute (19.14), (19.15) into (19.11) and then into (19.10)
and we note that as $\beta \rightarrow 0$ $R[1 - \underline{\beta} \cdot \hat{\underline{R}}] \sim R$

$$-\nabla V = \frac{q}{4\pi\epsilon_0} \frac{\hat{\underline{R}}}{R^2} + \frac{q}{4\pi\epsilon_0 c R} \left[\hat{\underline{R}} \times (\hat{\underline{R}} \times \dot{\underline{\beta}}) + \dot{\underline{\beta}} \right] \quad (19.16)$$

Exercise: verify these steps!

Reminder of expression
for A from earlier

$$\underline{A} = \beta A^0$$

$$\underline{A} = \frac{q}{4\pi\epsilon_0 c} \left[\frac{\underline{\beta}}{R(1 - \beta \cdot \hat{R})} \right]_{\text{ret}} \quad (19.8)$$

(19.7) & (19.8) are the Liénard - Wiechert
Potentials for a moving point charge

(Using 19.8)

$$-\frac{\partial \underline{A}}{\partial t} = -\frac{q}{4\pi\epsilon_0 c} \frac{\dot{\underline{\beta}}}{\underbrace{[R - \underline{\beta} \cdot \underline{R}]}_{\rightarrow R \text{ as } \beta \rightarrow 0}} + \underbrace{\underline{\beta} \frac{\partial \underline{A}}{\partial t}}_{\rightarrow 0 \text{ as } \beta \rightarrow 0}$$

$$= \frac{-q}{4\pi\epsilon_0 c R} \dot{\underline{\beta}}$$

(19.17)

Substitute (19.16) and (19.17) into (19.9) and re-insert "ret"

$$\underline{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{\hat{\underline{R}}}{R^2} + \frac{\hat{\underline{R}} \times (\hat{\underline{R}} \times \underline{\dot{\beta}})}{cR} \right]_{\text{ret}}$$

$$\underline{B} = \frac{1}{c} \left[\hat{\underline{R}} \cdot \right]_{\text{ret}} \times \underline{E}$$

Similarly show that

Simplified versions
of the
Liénard - Wiechert
Fields valid
for the case $\beta \rightarrow 0$