Continuation from Lecture 19

Vector Calculus.

Use (19.12) & (19.3)

$$-\underline{R} \times (\nabla \times \underline{\beta}) = \frac{1}{c} \underline{R} \times (\hat{\underline{R}} \times \dot{\underline{\beta}}) \qquad (19.14)$$

$$-(\underline{R} \cdot \nabla)\underline{\beta} = -x^i \frac{\partial}{\partial x^i} \underline{\beta} = -x^i \frac{\partial t_{ret}}{\partial x^i} \frac{\partial \underline{\beta}}{\partial t_{ret}}$$

$$= (\underline{R} \cdot \nabla t_{ret}) \dot{\underline{\beta}} = \frac{1}{c} (\underline{R} \cdot \hat{\underline{R}}) \dot{\underline{\beta}} = \frac{R}{c} \dot{\underline{\beta}} \qquad (19.15)$$

(using 19.13)

Substitute (19.14), (19.15) into (19.11) and then into (19.10)

and we note that as $\beta \to 0$ $R[1 - \underline{\beta} \cdot \hat{\underline{R}}] \sim R$

$$-\nabla V = \frac{q}{4\pi\varepsilon_0} \frac{\hat{\underline{R}}}{R^2} + \frac{q}{4\pi\varepsilon_0 c R} \left[\hat{\underline{R}} \times (\hat{\underline{R}} \times \dot{\underline{\beta}}) + \dot{\underline{\beta}} \right] \qquad (19.16)$$

Exercise: verify these steps!

$\boxed{\text{Reminder of expression for } \underline{A} \text{ from earlier}}$

$$\underline{A} = \underline{\beta} A^0$$

$$\underline{A} = \frac{q}{4\pi\varepsilon_0 c}\left[\frac{\underline{\beta}}{R(1-\underline{\beta}\cdot\hat{\underline{R}})}\right]_{ret} \qquad (19.8)$$

(19.7) & (19.8) are the <u>Liénard-Wiechert Potentials</u> for a moving point charge

(Using 19.8)

$$-\frac{\partial \underline{A}}{\partial t} = -\frac{q}{4\pi\varepsilon_0 c}\frac{\dot{\underline{\beta}}}{\underbrace{[R-\underline{\beta}.\underline{R}]}_{\to R \text{ as } \beta\to 0}} + \underbrace{\underline{\beta}\,\frac{\partial \underline{A}^{\circ}}{\partial t}}_{\to 0 \text{ as } \beta\to 0}$$

$$= \frac{-q}{4\pi\varepsilon_0 c R}\,\dot{\underline{\beta}} \qquad (19.17)$$

Substitute (19.16) and (19.17) into (19.9) and re-insert "ret"

$$\underline{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{\hat{\underline{R}}}{R^2} + \frac{\hat{\underline{R}} \times (\hat{\underline{R}} \times \dot{\underline{\beta}})}{cR}\right]_{ret}$$

Similarly show that

$$\underline{B} = \frac{1}{c}\left[\hat{\underline{R}}\right]_{ret} \times \underline{E}$$

Simplified versions of the <u>Liénard-Wiechert Fields</u> valid for the case $\beta \to 0$