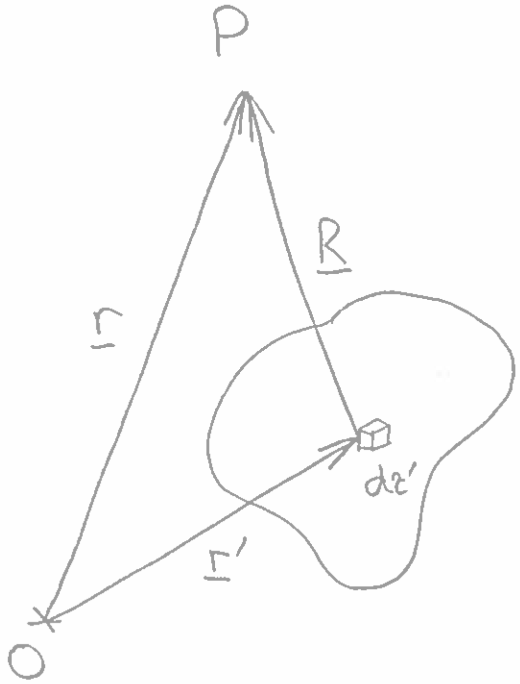
Summary from Lectures 1 and 2

In Electrostatics we have:



$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\mathbf{r}')}{R}\,d\tau' \qquad \text{where } \mathbf{R} = \mathbf{r} - \mathbf{r}'$$

We showed that $\nabla\left(\frac{1}{R}\right) = -\frac{\hat{\mathbf{R}}}{R^2}$

Therefore, defining $\mathbf{E} = -\nabla V$ gave

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{\mathbf{R}}}{R^2}\,\rho(\mathbf{r}')\,d\tau'$$

We showed that $\displaystyle \Phi = \oint_S \mathbf{E}\cdot d\mathbf{a} = \frac{1}{\varepsilon_0}\int_v \rho(\mathbf{r}')\,d\tau' = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$,

or in differential form $\displaystyle \nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$ or $\displaystyle \nabla^2 V = -\frac{\rho}{\varepsilon_0}$

Reminders

$\underline{r}'$ position of "source" (charge, current)

$\underline{r}$ position at which potential/field evalutated (relative to origin)

$\underline{R}$ position at which potential/field evaluated (relative to "source")

$$\nabla = \left(\hat{\underline{x}} \frac{\partial}{\partial x} + \hat{\underline{y}} \frac{\partial}{\partial y} + \hat{\underline{z}} \frac{\partial}{\partial z} \right)$$

represents partial differentiation w.r.t. the unprimed coordinates

$\therefore$ ∇ operating on any scalar $f(\underline{r}')$ or vector $\underline{v}(\underline{r}')$ that is a function only of the primed coordinates gives zero.