Lecture 21 summary: Radiation by point charges for $\beta \sim 1$

A) Total radiated power

$$\underbrace{P' = \frac{d\mathcal{E}'}{dt'}}_{\text{rest frame}} = \frac{\cancel{\gamma}\, d\mathcal{E}}{\cancel{\gamma}\, dt} = P \quad \text{is Lorentz invariant}$$

(because total momentum of radiation $= 0$)

Bremsstrahlung: $P = \dfrac{\mu_0 c q^2}{6\pi} \dot{\beta}^2 \gamma^6$

Synchrotron Radiation: $P = \dfrac{\mu_0 c q^2}{6\pi} \dot{\beta}^2 \gamma^4$

B) Angular distribution (general result)

$$\frac{dP'}{d\Omega'} = \gamma^4 (1 - \beta\cos\theta)^3 \frac{dP}{d\Omega}$$

Angular distribution for Bremsstrahlung

$$\frac{dP}{d\Omega} = \frac{\mu_0 c q^2}{16\pi^2} \frac{\sin^2\theta}{(1-\beta\cos\theta)^5} \dot{\beta}^2$$

Factor of $(1-\beta\cos\theta)^{-5}$ produces very sharp peak of radiation around direction of $\underline{\beta}$, $\underline{\dot{\beta}}$ as $\beta \rightarrow 1$

Angular distribution for Synchrotron Radiation

$$\frac{dP}{d\Omega} = \frac{\mu_0 c q^2}{16\pi^2} \frac{1}{(1-\beta\cos\theta)^3} \left(1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2}\right) \dot{\beta}^2$$

For $\beta \rightarrow 1$, sharp peak in direction of $\underline{\beta}$ (not $\underline{\dot{\beta}}$)