Summary of Lecture 3

Solutions to Laplace's equation $\nabla^2 V = 0$ have the following properties:

(A) Value of $V(r)$ is equal to average over symmetrically placed surrounding region

$\therefore$ (B) No local maxima or minima can occur within region satisfying $\nabla^2 V = 0$

$\because$ If, say, a minimum were present, we could define a region around it where all points had a higher value than at the minimum — this would violate our theorem (A) above.

$\Rightarrow$ Minima / maxima can occur at the boundary only.

(2) The solution to $\nabla^2 V = 0$ for a particular physical system requires knowledge of the boundary conditions.

(3) Uniqueness Theorem : Given a specific set of boundary conditions there is only one unique possible solution.

(a) specify V at boundary (Dirichlet)

(b) specify $\nabla V \cdot \hat{n}$, where $\hat{n}$ is $\perp$ surface at boundary (Neumann)