

General formalism for 2D (r, θ) problems

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) = 0 \quad (4.1)$$

Separation of variables:

$$V(r, \theta) = R(r) \Theta(\theta) \quad (4.2)$$

Exercise for you: combine (4.1) & (4.2) to obtain

$$\underbrace{\frac{1}{R} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right)}_{\text{constants } l(l+1)} + \underbrace{\frac{1}{\Theta \sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d\Theta}{d\theta} \right)}_{-l(l+1)} = 0 \quad (4.3)$$

State general solution $V(r, \theta)$

$$V(r, \theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

Exercise for
you:

Verify that

Solution to
equation for $R(r)$

$\Theta(\theta)$

$P_l(\cos \theta)$ are the Legendre Polynomials

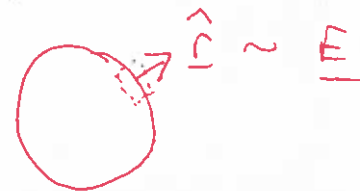
$$P_0(\cos \theta) = 1$$

$$P_1 = \cos \theta$$

$$P_2 = (3 \cos^2 \theta - 1) / 2$$

Exercises

1)
$$\underline{E} = -\nabla V = -\left(\frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta}\right)$$



2 Apply Gauss's law at surface of sphere to work out $\sigma(\theta)$