Summary of Lecture 5: Multipole Expansions in Electrostatics

Expanding in powers of $\frac{\underline{r}'}{\underline{r}}$ we can write

$$V(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\underline{r}')\,d\tau'}{R} = \frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{(n+1)}}\int (r')^n P_n(\cos\alpha)\,\rho(\underline{r}')\,d\tau',$$

where $P_n(\cos\alpha)$ are the <u>Legendre polynomials</u>.

The first few terms look like:

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left\{\frac{1}{r}\int \rho(\underline{r}')\,d\tau' + \frac{1}{r^2}\int r'\cos\alpha\,\rho(\underline{r}')\,d\tau' + \frac{1}{r^3}\int (r')^2\left[\frac{3}{2}\cos^2\alpha - \frac{1}{2}\right]\rho(\underline{r}')\,d\tau' + \dots\right\}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$ (Q is total charge)

"monopole" term

$$\frac{1}{4\pi\varepsilon_0}\frac{\hat{\underline{r}}\cdot\underline{p}}{r^2}$$

"dipole" term

$$\left[\underline{p} = \int \underline{r}'\rho(\underline{r}')\,d\tau' \text{ is the "}\underline{\text{dipole moment}}\text{"}\right]$$

"quadrupole" term