Summary of Lecture 9) Wave Equations for the Potentials and Solutions

Expressing Maxwell's equations in terms of the potentials and choosing to work in the "Lorenz gauge" $\boxed{\nabla \cdot \underline{A} = -\mu_0 \varepsilon_0 \frac{\partial V}{\partial t}}$,

we obtained the "inhomogeneous wave equations"

$$\boxed{\begin{aligned} \mu_0 \varepsilon_0 \frac{\partial^2 V}{\partial t^2} - \nabla^2 V &= \rho/\varepsilon_0 \\ \mu_0 \varepsilon_0 \frac{\partial^2 \underline{A}}{\partial t^2} - \nabla^2 \underline{A} &= \mu_0 \underline{j} \end{aligned}}$$

or

$$\boxed{\begin{aligned} \Box^2 V &= \frac{\rho}{\varepsilon_0} \\ \Box^2 \underline{A} &= \mu_0 \underline{j} \end{aligned} \quad \text{where} \quad \Box^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2}$$

We showed that the "retarded potentials" are solutions to these equations

$$\boxed{\begin{aligned} V(\underline{r}, t) &= \frac{1}{4\pi\varepsilon_0} \int_v \frac{\rho(\underline{r}', t_{ret})}{R} d\tau' \\ \underline{A}(\underline{r}, t) &= \frac{\mu_0}{4\pi} \int_v \frac{\underline{j}(\underline{r}', t_{ret})}{R} d\tau \end{aligned}}$$

, where ρ, $\underline{j}$ and R are evaluated at the "retarded time": $\boxed{t_{ret} = t - \frac{R}{c}}$ takes into account the time to propagate from $\underline{r}'$ to $\underline{r}$