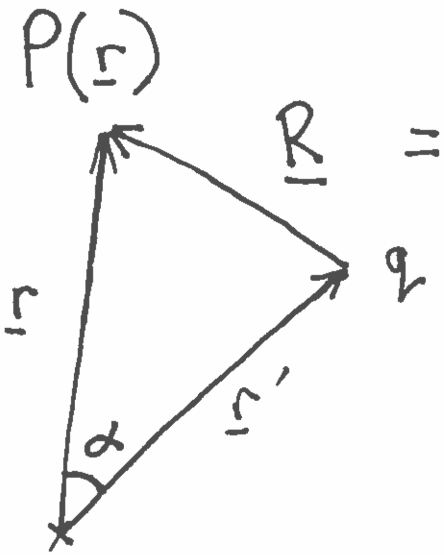
Lecture 1) Electrostatics

$V(\underline{r})$



$$\underline{R} = \underline{r} - \underline{r}' = (x-x')\hat{\underline{x}} + (y-y')\hat{\underline{y}} + (z-z')\hat{\underline{z}}$$

Unit vector $\hat{\underline{R}} = \dfrac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|} = \dfrac{\underline{R}}{|\underline{R}|} = \dfrac{\underline{R}}{R}$

Magnitude: $R = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{1/2}$

$$= \left[r^2 + r'^2 - 2rr'\cos\alpha\right]^{1/2}$$

<u>Define</u> $V(\underline{r}) = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{R}$

$$\nabla\left(\frac{1}{R}\right) = \left[\hat{\underline{x}}\frac{\partial}{\partial x} + \hat{\underline{y}}\frac{\partial}{\partial y} + \hat{\underline{z}}\frac{\partial}{\partial z}\right]\frac{1}{R}$$

$$= -\frac{\hat{\underline{R}}}{R^2}$$

[NB. x and x' are independent!

$\therefore \dfrac{\partial x'}{\partial x} = \dfrac{\partial y'}{\partial y} = \dfrac{\partial z'}{\partial z} = 0.$]

$$-\nabla V = \frac{-q}{4\pi\varepsilon_0}\nabla\left(\frac{1}{R}\right) = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\underline{R}}}{R^2} = \underline{E}(\underline{r})$$

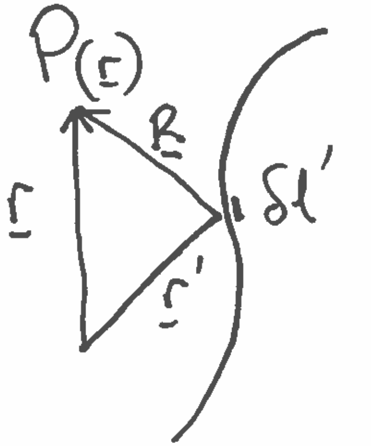
The Principle of Superposition

A) For a collection of point charges

$$V(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{R_i} \qquad \text{where } \underline{R}_i = \underline{r} - \underline{r}_i'$$

$$\underline{E}(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\sum_i q_i \frac{\hat{\underline{R}}_i}{R_i^2}$$

B) Continuous distributions of (static) charge

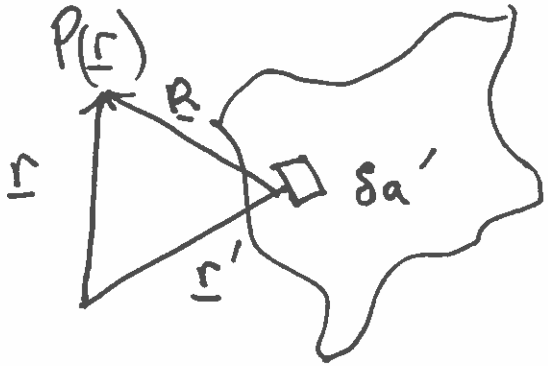


Line charge

$$dq' = \lambda(\underline{r}')\,dl'$$

$$V(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\underline{r}')\,dl'}{R}$$

$$\underline{E}(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{\underline{R}}}{R^2}\,\lambda(\underline{r}')\,dl'$$

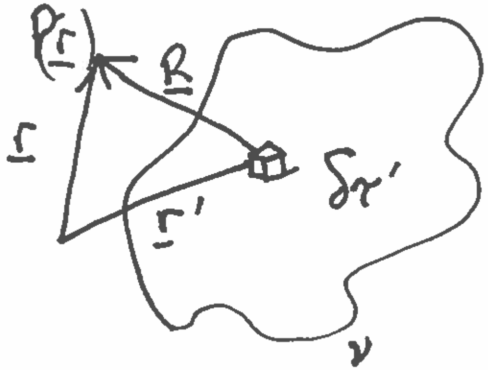


Surface charge

$$dq' = \sigma(\underline{r}')\,da'$$

$$V(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int_s \frac{\sigma(\underline{r}')\,da'}{R}$$

$$E(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int_s \frac{\hat{\underline{R}}}{R^2}\,\sigma(\underline{r}')\,da'$$



Volume charge

$$dq' = \rho(\underline{r}')\,d\tau'$$

$$V(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho(\underline{r}')\,d\tau'}{R}$$

$$E(\underline{r}) = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\hat{\underline{R}}}{R^2}\,\rho(\underline{r}')\,d\tau'$$