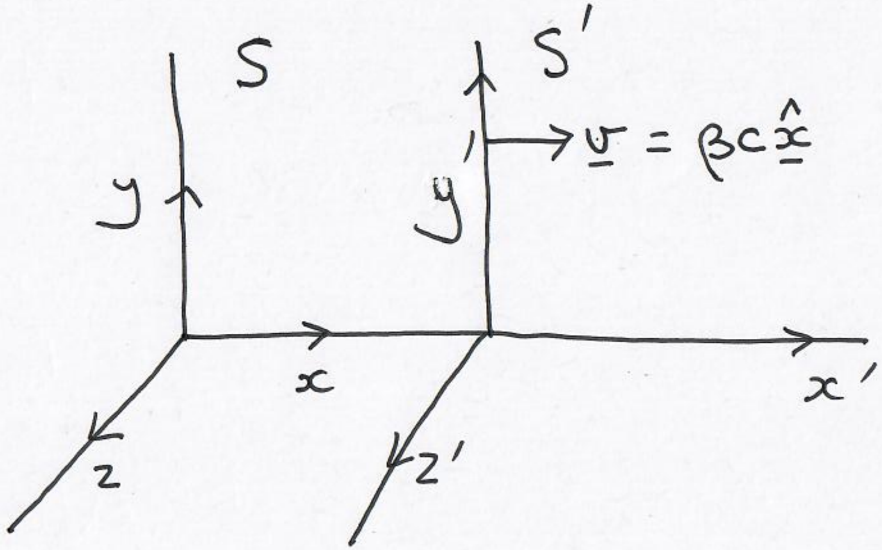
Lecture 10) Revision of Special Relativity

In an <u>inertial</u> frame of reference a body with zero nett force acting on it does not accelerate

Postulates

1) The laws of physics (results of experiments) are the same in all inertial frames of reference
2) The speed of light (in vacuum) is the same in all frames of reference
   - irrespective of the motion of the observer or the source.

Consider two inertial frames :



At times $t' = t = 0$, spatial coordinate axes and origins of S and S' coincide

Time and spatial coordinates in S and S' are related by the <u>Lorentz Transformations</u>

$$
\begin{aligned}
[ct'] &= \gamma\left([ct] - \beta x\right) \\
x' &= \gamma\left(-\beta[ct] + x\right) \\
y' &= y \\
z' &= z
\end{aligned}
$$

N.B. $[ct]$ has units of length

Renders expressions for Lorentz Transformation symmetric.

Central to our treatment of Special Relativity is the concept of a <u>4 - vector</u>

Archetypal example :

$$\underset{\sim}{x} = (ct, \underline{r}) = (ct, x, y, z) = (ct, x^i) = (x^0, x^1, x^2, x^3) = x^\mu$$

$i = 1, 2, 3$ $\qquad (\mu = 0, 1, 2, 3)$

illustrates the main properties of all 4-vectors

- has the structure (scalar, vector)
- for convenience we arrange for all 4 components to have same unit
- components transform according to the Lorentz Transformation

$$x'^0 = \gamma(x^0 - \beta x^1), \quad x'^1 = \gamma(-\beta x^0 + x^1), \quad x'^2 = x^2, \; x'^3 = x^3$$

$\underset{\sim}{x}(1) - \underset{\sim}{x}(2) = \Delta \underset{\sim}{x} = (c\Delta t, \Delta \underline{r})$ also transforms as a 4-vector

<u>Exercise for you</u>: verify this.

Scalar product of two 4-vectors

$$\underset{\sim}{a} \cdot \underset{\sim}{b} \equiv a^0 b^0 - \underline{a} \cdot \underline{b} = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

has the same value in all inertial frames

$$\therefore \quad \underset{\sim}{a}' \cdot \underset{\sim}{b}' = \underset{\sim}{a} \cdot \underset{\sim}{b}$$

or "is invariant with respect to a Lorentz transformation" or "is Lorentz invariant" or "is a Lorentz scalar"

<u>Exercise for you</u>: verify this.

The "<u>magnitude</u>" of a 4-vector is just a special case

$$\underset{\sim}{a}' \cdot \underset{\sim}{a}' = \underset{\sim}{a} \cdot \underset{\sim}{a} \qquad (\text{is invariant})$$

Example of using the scalar product

Consider the space-time interval between two events in S connected by a signal travelling at the speed of light.

Thus: $\dfrac{\Delta r}{\Delta t} = c$

$$\therefore \left(\Delta \underset{\sim}{x}\right)^2 = (c\Delta t)^2 - (\Delta r)^2 = 0$$

$$\left(\Delta \underset{\sim}{x}'\right) = 0 = (c\Delta t')^2 - (\Delta r')^2 = 0 \quad \therefore \frac{\Delta r'}{\Delta t'} = c$$

$\therefore$ Signal travels at speed of light in frame S'

Exercise for you: prove this result alternatively using the Lorentz transformation.

Additional exercises provided on the Special Relativity Revision sheet.