

Putting all this together we have

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{matrix} \rightarrow \nu \\ \downarrow \mu \end{matrix} \begin{bmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{bmatrix}$$

The "electromagnetic field tensor"

Exercise: Verify explicitly each element of $F^{\mu\nu}$