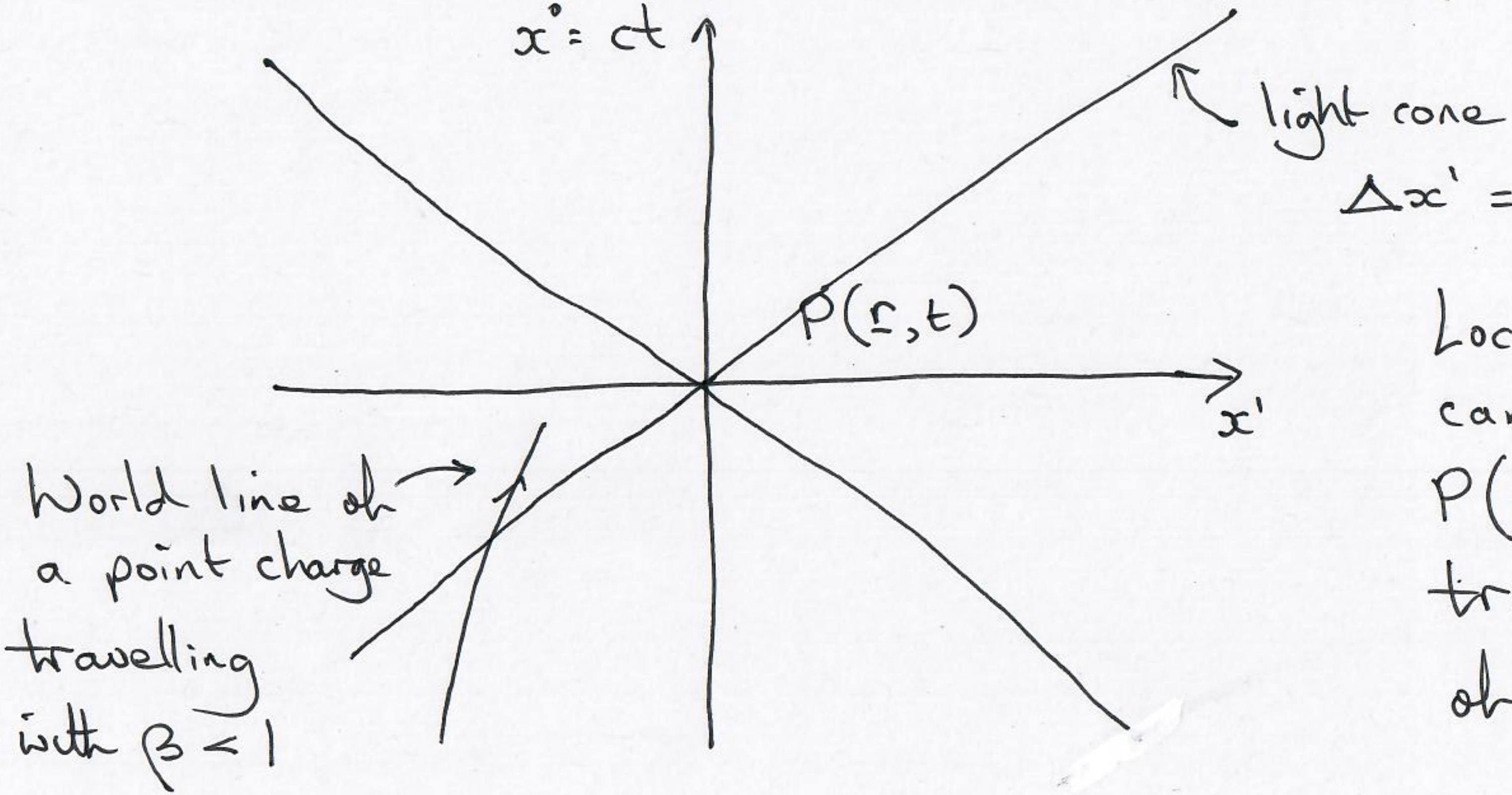
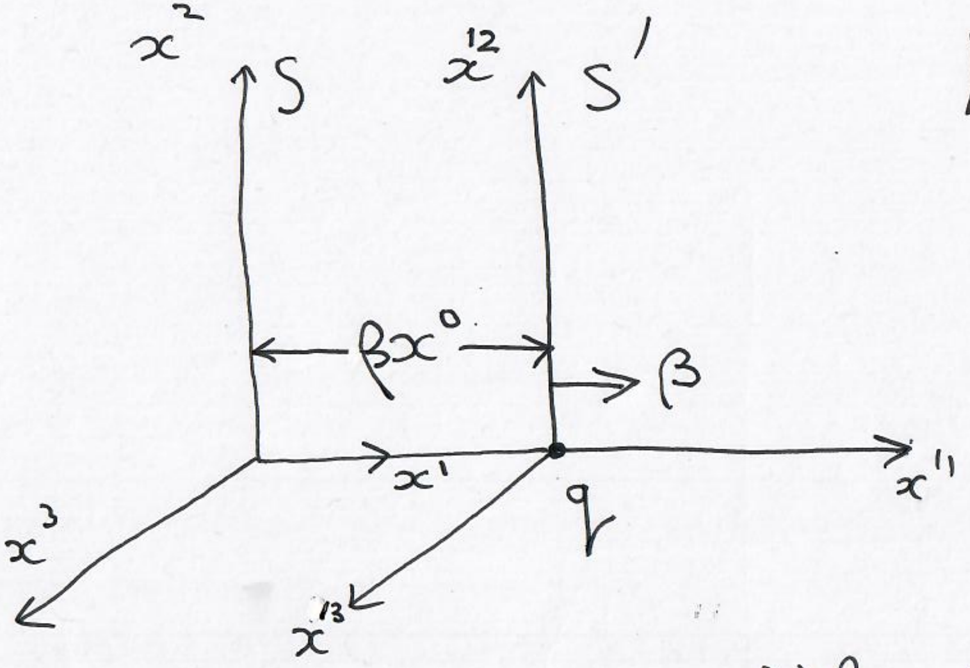
Lecture 14) Potentials for a point charge, q, moving with a constant velocity $\underline{\beta} = \beta\,\hat{\underline{x}}^1$



$$\Delta x^1 = c\,\Delta t = \Delta x^0$$

Locus of all points that can be connected with $P(\underline{r}, t)$ by a signal travelling at the speed of light, c.

Note that world line can cross the <u>past</u> light cone of $P(\underline{r}, t)$ at one and only one point in space-time.



Let charge q be at rest at the origin of frame S'.

We want to evaluate the potentials A^μ in frame S in which the charge is moving with $\underline{\beta} = \beta \hat{\underline{x}}^1$

N.B. Need to transform A'^μ <u>but also</u>

$$x'^1 = \gamma\left[x^1 - \beta x^0\right] \quad !$$

In rest frame S':

$$A'^0 = \frac{V'}{c} = \frac{q}{4\pi\varepsilon_0 c}\frac{1}{R'}, \quad \underline{A}' = 0$$

where $(R')^2 = (x'^1)^2 + (x'^2)^2 + (x'^3)^2 = \left(\gamma\left[x^1 - \beta x^0\right]\right)^2 + (x^2)^2 + (x^3)^2$

Potentials in frame S

L.T. $$\frac{V}{c} = A^0 = \gamma(A'^0 + \beta A'^1) = \gamma A'^0$$ (Note: inverse Lorentz Transformation.)

$$\therefore \quad A^0 = \frac{q}{4\pi\varepsilon_0 c}\,\gamma\,\frac{1}{\left[\left(\gamma[x^1-\beta x^0]\right)^2+(x^2)^2+(x^3)^2\right]^{1/2}}$$

$$A^1 = \gamma(\beta A'^0 + A'^1) = \gamma\beta A'^0 = \beta A^0$$

$$= \frac{q}{4\pi\varepsilon_0 c}\,\gamma\beta\,\frac{1}{\left[\left(\gamma[x^1-\beta x^0]\right)^2+(x^2)^2+(x^3)^2\right]^{1/2}}$$

$$A^2 = A^3 = 0$$

Exercise for you: Cross-check that $A'^{\mu}A'_{\mu} = A^{\mu}A_{\mu}$.

What do these potentials "look like"?

Along the x^1 axis: $x^2 = x^3 = 0$: $A^0 = \frac{V}{c} = \frac{q}{4\pi\varepsilon_0 c} \gamma \frac{1}{\gamma |x^1 - \beta x^0|}$

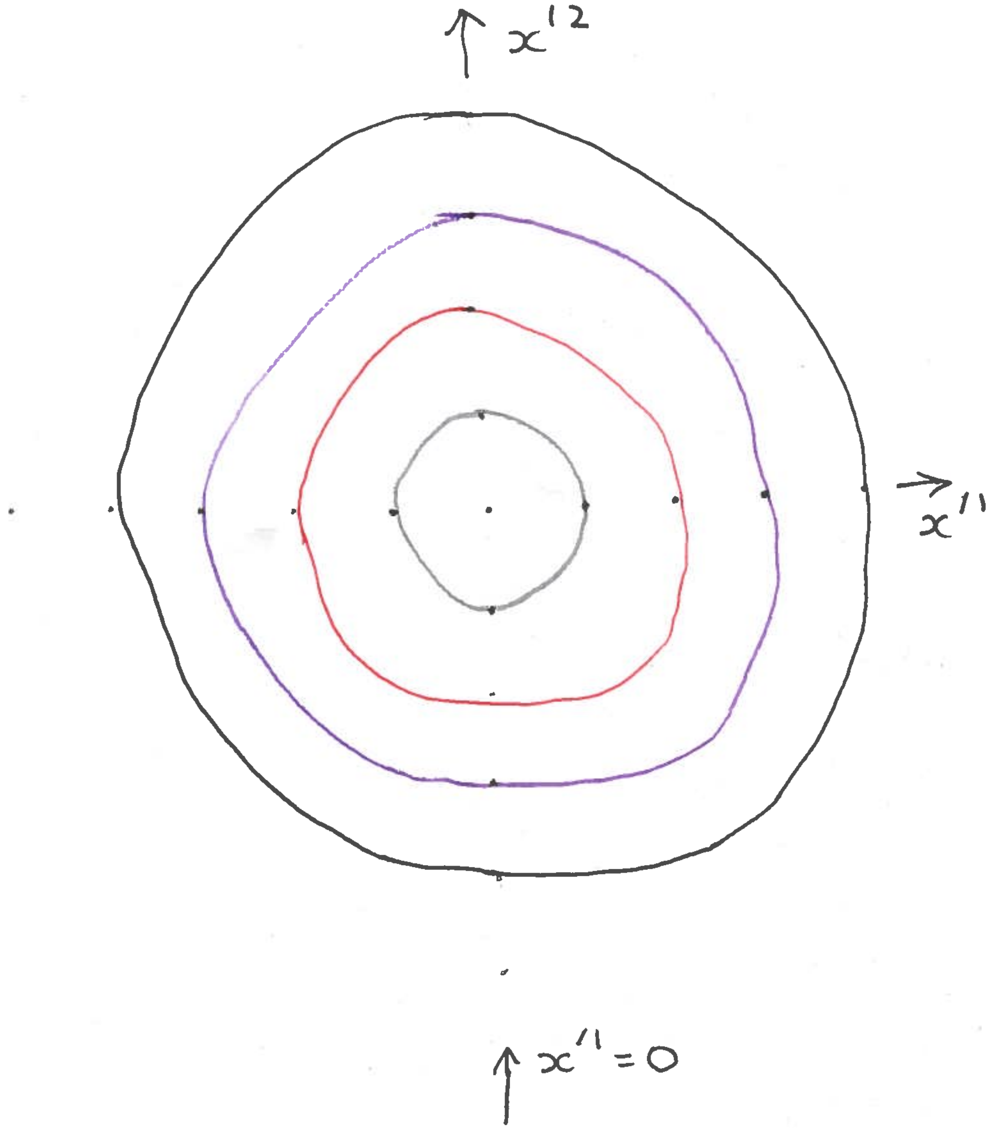
That is, the same potential as for a stationary charge q_0 at the "current" position $x^1 = \beta x^0$, $x^2 = x^3 = 0$!

Transverse to x^1 axis at the current position $x^1 = \beta x^0$ of the charge:

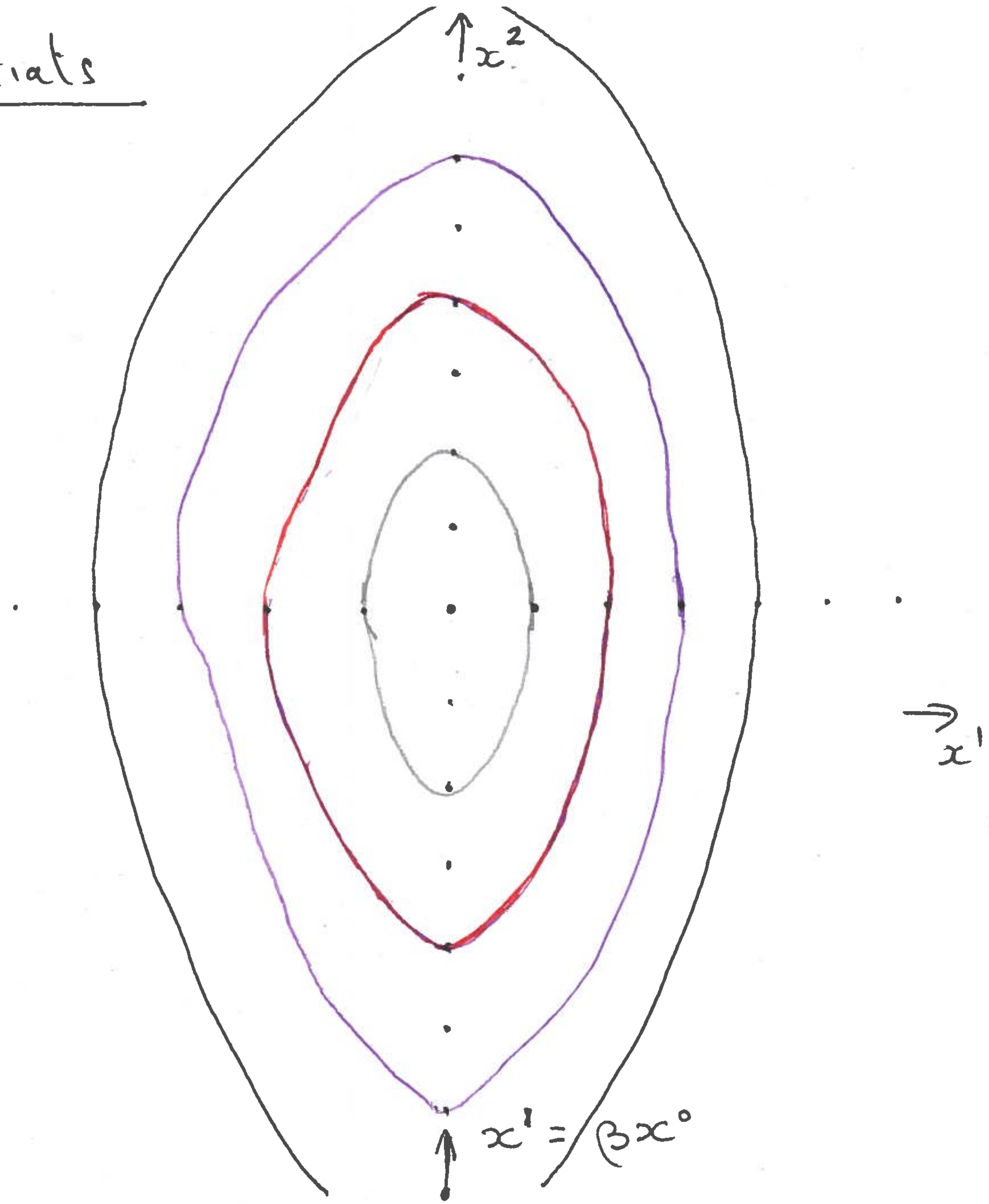
$x^1 - \beta x^0 = 0$: $A^0 = \frac{V}{c} = \gamma \left[\frac{q}{4\pi\varepsilon_0 c} \frac{1}{[(x^2)^2 + (x^3)^2]^{1/2}} \right]$

That is γ times the potential for a stationary charge at $x^1 = \beta x^0$, $x^2 = x^3 = 0$.

Sketches of the equipotentials



Stationary charge



Moving charge with $\gamma \sim 2$.

Perhaps rather surprisingly, these equipotentials are centred at the "current" position $x' = \beta x^0$ of the charge, and not at the position of the charge at the retarded time (which is where the electromagnetic signal was sent from to arrive at $P(\underline{r}, t)$!?