Potentials in frame S

$\underline{\text{L.T.}}$ $$\frac{V}{c} = A^{0} = \gamma\left(A'^{0} + \beta A'^{1}\right) = \gamma A'^{0}$$ (Note: inverse Lorentz Transformation.)

$$\therefore A^{0} = \frac{q}{4\pi\varepsilon_0 c}\,\gamma\,\frac{1}{\left[\left(\gamma\left[x^{1}-\beta x^{0}\right]\right)^{2}+\left(x^{2}\right)^{2}+\left(x^{3}\right)^{2}\right]^{1/2}}$$

$$A^{1} = \gamma\left(\beta A'^{0} + A'^{1}\right) = \gamma\beta A'^{0} = \beta A^{0}$$

$$= \frac{q}{4\pi\varepsilon_0 c}\,\gamma\beta\,\frac{1}{\left[\left(\gamma\left[x^{1}-\beta x^{0}\right]\right)^{2}+\left(x^{2}\right)^{2}+\left(x^{3}\right)^{2}\right]^{1/2}}$$

$$A^{2} = A^{3} = 0$$

Exercise for you: Cross-check that $A'^{\mu}A'_{\mu} = A^{\mu}A_{\mu}$.