For $\nu = 1$

$$\partial_\mu F^{\mu 1} = \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{-E_1}{c}\right) + 0 + \frac{\partial (B_3)}{\partial x^2} + \frac{\partial}{\partial x^3}\left(-B_2\right)$$

$$= \left[-\frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} + \nabla \times \underline{B} \right]_1 = \mu_0 j^1$$

Exercise: repeat for the x^2 and x^3 components.

$$\therefore \quad \nabla \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

An Exercise for you

Show that

$$\boxed{\partial^\mu F^{\nu\lambda} + \partial^\lambda F^{\mu\nu} + \partial^\nu F^{\lambda\mu} = 0}$$

correspond to the two homogeneous Maxwell's Equations

(Note the cyclic permutation of indices.)

Hint: try using $\left.\begin{array}{l} 1, 2, 3 \\ \text{and also} \quad 0, 1, 2 \end{array}\right\}$ for μ, ν, λ

Let's start with $\mu = 0$:

$$q F^{0\nu} u_\nu = q\left[0 + \left(-\frac{E_1}{c}\right)\left(-\gamma u_1\right) + \left(-\frac{E_2}{c}\right)\left(-\gamma u_2\right) + \left(\frac{E_3}{c}\right)\left(-\gamma u_3\right)\right]$$

$$= q\gamma \frac{\underline{u}\cdot\underline{E}}{c}$$

For $\mu = 1$:

$$q F^{1\nu} u_\nu = q\left[\frac{E_1}{c}\gamma c + 0 + \left(-B_3\right)\left(-\gamma u_2\right) + B_2\left(-\gamma u_3\right)\right]$$

$$= q\gamma\left[\underline{E} + \underline{u}\times\underline{B}\right]_1$$

Exercise: repeat for the x^2 and x^3 components

$$\therefore\ q F^{\mu\nu} u_\nu = q\gamma\left(\frac{\underline{u}\cdot\underline{E}}{c}, \underline{E} + \underline{u}\times\underline{B}\right)$$

Exercise: obtain the same result by multiplying matrices $F^{\mu\nu} u_\nu$.

The Interaction of charged particles with the fields

$$\boxed{\frac{dp^\mu}{d\tau} = q\, F^{\mu\nu}\, u_\nu}$$

proper time maintains left hand side as a 4-vector.

right hand side is also a 4-vector.

Exercise for you

Show that

$$\frac{d\underline{p}}{dt} = q\left(\underline{E} + \underline{u} \times \underline{B}\right)$$ the Lorentz force law

can be obtained from the $\mu = 1, 2, 3$ components of our 4-vector $\frac{dp^\mu}{d\tau}$.