Lecture 19(b) Use vector calculus to find $\underline{E}$, $\underline{B}$ produced by an accelerating point charge

$$\underline{E} = -\nabla V - \frac{\partial \underline{A}}{\partial t} \qquad (19.9)$$

Just for brevity we'll drop the "ret" when not strictly needed and put it back in at the end.

Using (19.7)

$$-\nabla V = -c\nabla A^0 = -\frac{q}{4\pi\varepsilon_0} \frac{-1}{[R - \underline{\beta}.\underline{R}]^2} \left(\nabla R - \nabla(\underline{\beta}.\underline{R})\right) \qquad (19.10)$$

Exercise for you: show that $\nabla R = \hat{\underline{R}}$.

Using identity (4) from the vector sheet

$$-\nabla(\underline{\beta}.\underline{R}) = -\underbrace{\underline{\beta} \times (\nabla \times \underline{R})}_{\sim 0 \text{ as } \beta \to 0} - \underline{R} \times (\nabla \times \underline{\beta}) - \underbrace{(\underline{\beta}.\nabla)\underline{R}}_{\sim 0 \text{ as } \beta \to 0} - (\underline{R}.\nabla)\underline{\beta} \qquad (19.11)$$

$$\nabla \times \underline{\beta} = \varepsilon_{ijk} \frac{\partial}{\partial x^j} \beta_k = \varepsilon_{ijk} \frac{\partial t_{ret}}{\partial x^j} \frac{\partial \beta_k}{\partial t_{ret}} = \nabla t_{ret} \times \dot{\underline{\beta}} \quad (19.1$$

$$t_{ret} = t - \frac{R}{c} \underset{\beta \to 0}{\Longrightarrow} \nabla t_{ret} = -\frac{\nabla R}{c} = -\frac{\hat{\underline{R}}}{c} \quad (19.13)$$

Using (19.12) and (19.13)

$$-\underline{R} \times (\nabla \times \underline{\beta}) = \frac{1}{c} \underline{R} \times (\hat{\underline{R}} \times \dot{\underline{\beta}}) \quad (19.14)$$

$$-(\underline{R}.\nabla)\underline{\beta} = -x^i \frac{\partial}{\partial x^i} \underline{\beta} = -x^i \frac{\partial t_{ret}}{\partial x^i} \frac{\partial \underline{\beta}}{\partial t_{ret}}$$

$$= -(\underline{R}.\nabla t_{ret}) \dot{\underline{\beta}} = \frac{1}{c}(\underline{R}.\hat{\underline{R}}) \dot{\underline{\beta}} = \frac{R}{c} \dot{\underline{\beta}} \quad (19.15)$$

(using 19.3)

N.B. We <u>define</u> $\dot{\underline{\beta}} = \frac{\partial \underline{\beta}}{\partial t_{ret}}$. (See note appended to lecture.)

Substitute (19.14) and (19.15) into (19.11) and then into (19.10) also noting that as $\beta \to 0$ $[R - \underline{\beta}.\underline{R}] \sim R$

$$-\nabla V = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\underline{R}}}{R^2} + \frac{q}{4\pi\varepsilon_0 c R}\left[\hat{\underline{R}} \times (\hat{\underline{R}} \times \dot{\underline{\beta}}) + \dot{\underline{\beta}}\right] \qquad (19.16)$$

~~Ex~~ercise for you confirm this last step.

Using (19.8)

$$-\frac{\partial \underline{A}}{\partial t} = \frac{-q}{4\pi\varepsilon_0 c}\underbrace{\left[\frac{\dot{\underline{\beta}}}{[R - \underline{\beta}.\underline{R}]}\right]}_{\to \frac{\dot{\underline{\beta}}}{R} \text{ as } \beta \to 0} + \underbrace{\underline{\beta}\frac{\partial A^{\circ}}{\partial t}}_{\sim 0 \text{ as } \beta \to 0}$$

$$-\frac{\partial \underline{A}}{\partial t} = \frac{-q}{4\pi\varepsilon_0 c R}\dot{\underline{\beta}} \qquad (19.17)$$

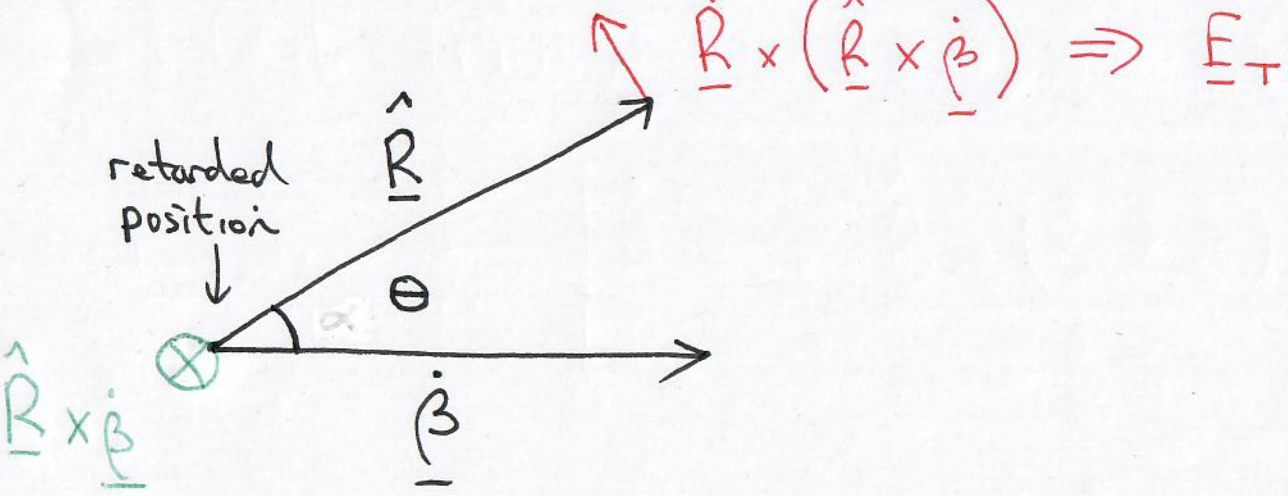
Substitute (19.16) and (19.17) into (19.9)

$$\underline{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{\hat{\underline{R}}}{R^2} + \frac{\hat{\underline{R}} \times (\hat{\underline{R}} \times \dot{\underline{\beta}})}{cR}\right]_{ret}$$

Similarly can show that

$$\underline{B} = \frac{1}{c}\left[\hat{\underline{R}}\right]_{ret} \times \underline{E}$$

Simplified version of the <u>Liénard - Wiechert</u> fields valid for the case $\beta \to 0$



$\therefore$ Tangential component of $\underline{E}$

$$E_T = \frac{q}{4\pi\varepsilon_0 c} \frac{\dot{\beta} \sin\theta}{R}$$

As found in Lecture 18) following our intuitive approach.