Lecture 19(b) Use vector calculus to find $\underline{E}$, $\underline{B}$ produced by an accelerating point charge

$$\underline{E} = -\nabla V - \frac{\partial \underline{A}}{\partial t} \qquad (19.9)$$

Just for brevity we'll drop the "ret" when not strictly needed and put it back in at the end.

Using (19.7)

$$-\nabla V = -c\nabla A^0 = \frac{-q}{4\pi\varepsilon_0} \frac{-1}{[R-\underline{\beta}.\underline{R}]^2}\left(\nabla R - \nabla(\underline{\beta}.\underline{R})\right) \qquad (19.10)$$

Exercise for you: show that $\nabla R = \hat{\underline{R}}$.

Using identity (4) from the vector sheet

$$-\nabla(\underline{\beta}.\underline{R}) = -\underbrace{\underline{\beta}\times(\nabla\times\underline{R})}_{\sim 0 \text{ as } \beta\to 0} - \underline{R}\times(\nabla\times\underline{\beta}) - \underbrace{(\underline{\beta}.\nabla)\underline{R}}_{\sim 0 \text{ as } \beta\to 0} - (\underline{R}.\nabla)\underline{\beta} \qquad (19.11)$$

Substitute (19.14) and (19.15) into (19.11) and then into (19.10) also noting that as $\beta \to 0$ $[R - \underline{\beta}.\underline{R}] \sim R$

$$-\nabla V = \frac{q}{4\pi\varepsilon_0} \frac{\hat{\underline{R}}}{R^2} + \frac{q}{4\pi\varepsilon_0 c R} \left[\hat{\underline{R}} \times \left(\hat{\underline{R}} \times \dot{\underline{\beta}} \right) + \dot{\underline{\beta}} \right] \qquad (19.16)$$

Exercise for you confirm this last step.

Using (19.18)

$$-\frac{\partial \underline{A}}{\partial t} = \underbrace{\frac{-q}{4\pi\varepsilon_0 c} \left[\frac{\dot{\underline{\beta}}}{[R - \underline{\beta}.\underline{R}]} \right]}_{\to \frac{\dot{\underline{\beta}}}{R} \text{ as } \beta \to 0} + \underbrace{\underline{\beta} \frac{\partial A^\circ}{\partial t}}_{\sim 0 \text{ as } \beta \to 0}$$

$$-\frac{\partial \underline{A}}{\partial t} = \frac{-q}{4\pi\varepsilon_0 c R} \dot{\underline{\beta}} \qquad (19.17)$$