$$\nabla R = \left(\hat{\underline{x}} \frac{\partial}{\partial x} + \hat{\underline{y}} \frac{\partial}{\partial y} + \hat{\underline{z}} \frac{\partial}{\partial z} \right) \left(x^2 + y^2 + z^2 \right)^{1/2}$$

$$= \frac{1}{2} \frac{1}{\left(x^2+y^2+z^2\right)^{1/2}} \left[\hat{\underline{x}}(2x) + \hat{\underline{y}}(2y) + \hat{\underline{z}}(2z) \right]$$

$$= \frac{\underline{R}}{R} = \hat{\underline{R}}$$

or, just to practice index notation

$$\nabla R = \hat{\underline{x}}_i \frac{\partial}{\partial x_i} \left(x_j x_j \right)^{1/2} = \frac{1}{2} \frac{1}{\left(x_k x_k\right)^{1/2}} \left(\hat{\underline{x}}_i \, 2 x_j \delta_{ij} \right) = \frac{\underline{x}_i x_i}{R} = \frac{\underline{R}}{R} = \hat{\underline{R}}$$

Note: use k here in order
not to get confused with j here

Substituting (19.14) and (19.15) into (19.11) gives

$$-\nabla\left(\underline{\beta}\cdot\underline{R}\right) = \frac{1}{c}\,\underline{R}\times\left(\hat{\underline{R}}\times\dot{\underline{\beta}}\right) + \frac{R}{c}\,\dot{\underline{\beta}} \qquad \text{(Eqn A)}$$

Setting $\left[R - \underline{\beta}\cdot\underline{R}\right] = R$ in (19.10) gives

$$-\nabla V = \frac{q}{4\pi\varepsilon_0 R^2}\left(\nabla R - \nabla\left(\underline{\beta}\cdot\underline{R}\right)\right)$$

Substituting (A) into this equation and using $\nabla R = \hat{\underline{R}}$ gives

$$-\nabla V = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\underline{R}}}{R^2} + \frac{q}{4\pi\varepsilon_0 c R}\left[\hat{\underline{R}}\times\left(\hat{\underline{R}}\times\dot{\underline{\beta}}\right) + \dot{\underline{\beta}}\right] \qquad (19.16)$$

as required.