Define $V(\underline{r}) = \frac{q}{4\pi\varepsilon_0}\frac{1}{R}$

$$\nabla\left(\frac{1}{R}\right) = \left[\hat{\underline{x}}\frac{\partial}{\partial x} + \hat{\underline{y}}\frac{\partial}{\partial y} + \hat{\underline{z}}\frac{\partial}{\partial z}\right]\frac{1}{R}$$

Exercise for you: Insert explicit expression for R in cartesian coordinates and derive the given result.

$$= -\frac{\hat{\underline{R}}}{R^2}$$

[NB. x and x' are independent! $\therefore \frac{\partial x'}{\partial x} = \frac{\partial y'}{\partial y} = \frac{\partial z'}{\partial z} = 0$.]