Define $V(\underline{r}) = \frac{q}{4\pi\varepsilon_0}\frac{1}{R}$

$$\nabla\left(\frac{1}{R}\right) = \left[\hat{\underline{x}}\frac{\partial}{\partial x} + \hat{\underline{y}}\frac{\partial}{\partial y} + \hat{\underline{z}}\frac{\partial}{\partial z}\right]\frac{1}{R}$$

$$= -\frac{1}{2}\frac{\left[2(x-x')\hat{\underline{x}} + 2(y-y')\hat{\underline{y}} + 2(z-z')\hat{\underline{z}}\right]}{\left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{3/2}}$$

$$= -\frac{\underline{R}}{R^3}$$

$$= -\frac{\hat{\underline{R}}}{R^2}$$

[NB. x and x' are independent!
$\therefore \frac{\partial x'}{\partial x} = \frac{\partial y'}{\partial y} = \frac{\partial z'}{\partial z} = 0.$]