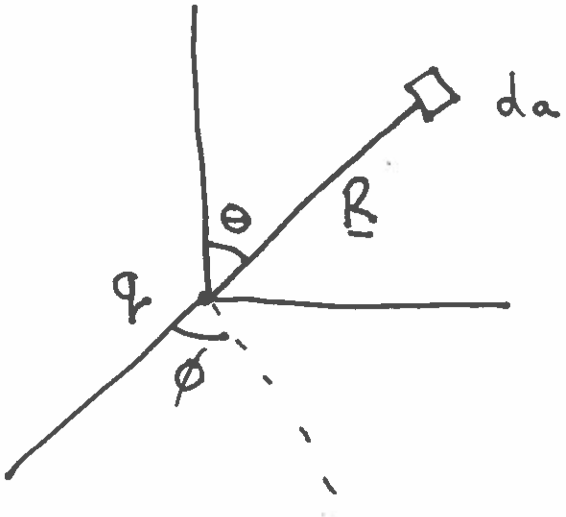
Lecture 2) Electrostatics (part 2)

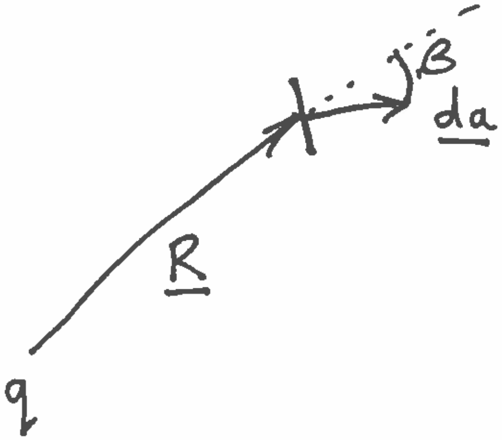
Since $\underline{E} = -\nabla V$

then $\nabla \times \underline{E} = -\nabla \times (\nabla V) = 0$

from vector identity number 10
(see the sheet of useful formulae posted on the course web-site).

Φ : The flux of $\underline{E}$ through any arbitrary closed surface S that encloses a point charge q





$$da = \frac{R^2 \sin\theta \, d\theta \, d\phi}{\cos\beta}$$

Without loss of generality we place q at the centre of our coordinate system as long as R and β can vary arbitrarily with θ and ϕ

$$d\Phi = \underline{E} \cdot \underline{da} = E \, da \cos\beta$$

$$= \frac{q}{4\pi\varepsilon_0} \sin\theta \, d\theta \, d\phi$$

i.e. Independent of R and β
Depends only on solid angle subtended

Exercise for you: Integrate over the entire solid angle in θ and ϕ to obtain the given result.

$$\Phi = \frac{q}{\varepsilon_0}.$$

From the principle of superposition this result generalizes to multiple point charges q_i enclosed by S

$$\Phi = \frac{1}{\varepsilon_0} \sum_i q_i$$

or for a continuous charge we can do a volume integral.

$$\frac{1}{\varepsilon_0} \int_v \rho(\underline{r}')\, d\tau' = \Phi = \oint_s \underline{E} \cdot \underline{da} = \int_v \nabla \cdot \underline{E}\, d\tau'$$

where v is the volume enclosed by S

True for all v only if $\nabla \cdot \underline{E} = \rho / \varepsilon_0$

In terms of V

$$\nabla \cdot \underline{E} = \nabla \cdot (-\nabla V) = -\nabla^2 V$$

$$\therefore \quad \nabla^2 V = -\rho/\varepsilon_0 \qquad \text{Poisson's Equation}$$

In regions free of charges

$$\nabla^2 V = 0 \qquad \text{Laplace's Equation}$$

2nd order differential equation

To completely specify solutions requires some boundary conditions.