Lecture 21 (a) Radiation by point charges moving with relativistic speeds (part 1)

Starting with the results from Lecture 20, perform Lorentz transformations of P (Eqn 20.4) and $\frac{dP}{d\Omega}$ (Eqn 20.3) from the rest frame, S', of an accelerating point charge to frame S, in which it is moving with speed β (in units of c) along the x' axis.

Patterns of radiation produced by an accelerating point charge

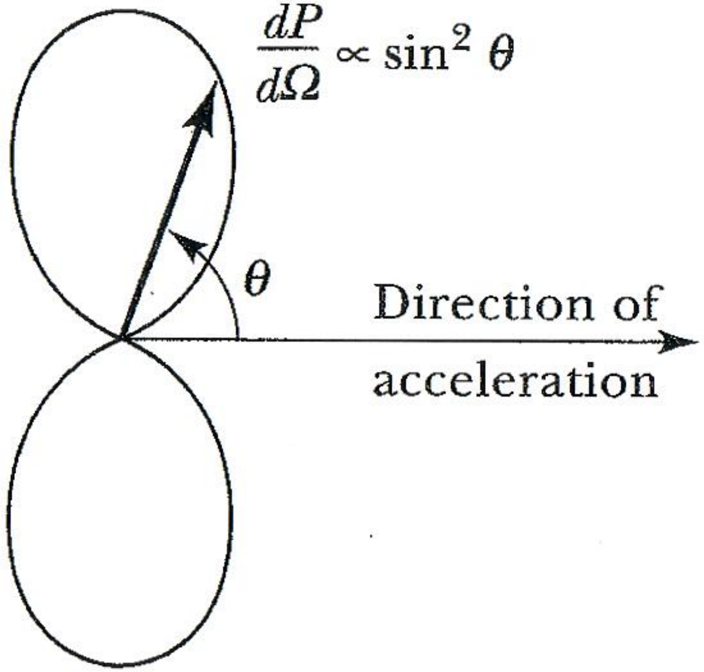

Figure 1: Angular dependence of (Larmor) radiation from a slowly moving accelerating charge $\beta \ll 1$ [1].

Radiation for the case $\beta \ll 1$

From Lecture 19(b) we had

From Lecture 20

$$\underline{E} = \frac{q}{4\pi\varepsilon_0} \frac{\sin\theta \, \dot{\beta}}{cR}$$

$$\underline{S} = \frac{1}{\mu_0 c} E^2 \left[\hat{\underline{R}}\right]_{ret} = \frac{1}{\mu_0 c} \frac{q^2}{16\pi^2\varepsilon_0^2} \frac{\sin^2\theta \, \dot{\beta}^2}{c^2 R^2} \left[\hat{\underline{R}}\right]_{ret}$$

$$= \frac{\mu_0 c \, q^2}{16\pi^2} \frac{\sin^2\theta \, \dot{\beta}^2}{R^2} \left[\hat{\underline{R}}\right]_{ret} \qquad (20.2)$$

S gives the radiated power per unit area

$R^2 S$ gives power per unit solid angle

$$\boxed{\frac{dP}{d\Omega} = \frac{\mu_0 c \, q^2}{16\pi^2} \sin^2\theta \, \dot{\beta}^2 \left[\hat{\underline{R}}\right]_{ret} \qquad (20.3)}$$

which is independent of R - as expected.

Total rate with which energy is radiated (for $\beta \ll 1$)

$$P = \int \frac{dP}{d\Omega}\, d\Omega = \frac{\mu_0 c q^2 \dot{\beta}^2}{16\pi^2} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^{\pi} \sin^2\theta \sin\theta \, d\theta}$$

Exercise for you: Make substitution $u = \cos\theta$ to show integral $= \frac{4}{3}$

$$\boxed{P = \frac{\mu_0 c q^2 \dot{\beta}^2}{6\pi}}$$

"The Larmor Formula" (20.4)

From Lecture 20

A) Total Radiated Power

$$P' = \frac{d\mathcal{E}'}{dt'} = \frac{\mu_0 c q^2}{6\pi}\left(\dot{\beta}'\right)^2 \qquad \text{(Eqn 20.4)}$$

transform this

Radiated energy : $d\mathcal{E} = \gamma\left(d\mathcal{E}' + 0\right)$

In S' the radiation has zero total momentum

Time interval : $dt = \gamma\left(dt' + 0\right)$

proper time

In S' charge is at rest.

$\therefore \quad P = P'$

Total power (left-hand side of expression) is Lorentz invariant.

Consider two special cases

1) $\underline{\beta} \parallel \underline{\dot{\beta}}$ $\qquad \dot{\beta}' = \gamma^3 \dot{\beta}$ (See Q2 of examples sheet 2)

"Bremsstrahlung"

$$P = \frac{\mu_0 c q^2}{6\pi} \dot{\beta}^2 \gamma^6 \qquad (21.1)$$

2) $\underline{\beta} \perp \underline{\dot{\beta}}$ $\qquad \dot{\beta}' = \gamma^2 \dot{\beta}$

"Synchrotron radiation"

$$P = \frac{\mu_0 c q^2}{6\pi} \dot{\beta}^2 \gamma^4 \qquad (21.2)$$

Do these factors of γ make intuitive sense?

$\dot{\beta}'$ measured in the rest frame : "proper acceleration"

$$\dot{\beta}'_{\perp} \sim \frac{d^2(x'^2)}{(dt')^2} \sim \frac{d^2 x^2}{(dt/\gamma)^2} = \gamma^2 \frac{d^2 x^2}{dt^2} = \gamma^2 \dot{\beta}_{\perp}$$

$$\dot{\beta}'_{\parallel} \sim \frac{d^2(x'^1)}{(dt')^2} \sim \frac{d^2(\gamma x^1)}{(dt/\gamma)^2} = \gamma^3 \frac{d^2 x^1}{dt^2} = \gamma^3 \dot{\beta}_{\parallel}$$