Lecture 21(b) Angular distribution of radiation by accelerating point charge moving with relativisitic speeds

B) Angular Distributions

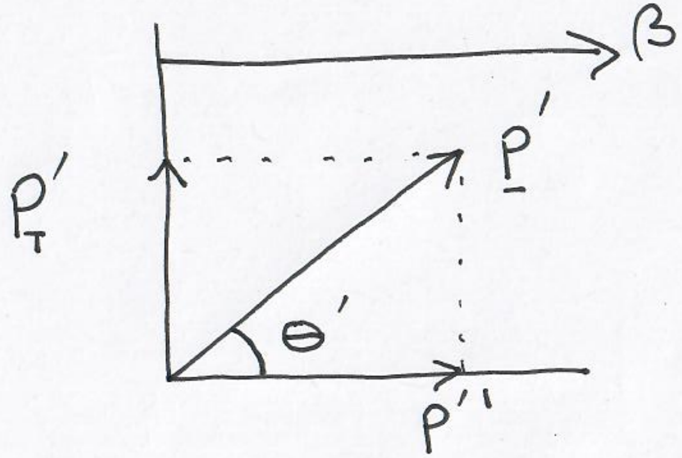
Transforming $\frac{dP'}{d\Omega'}$ is a bit more complicated!

$$\frac{dP'}{d\Omega'} = \frac{d}{d\Omega'}\left(\frac{d\mathcal{E}'}{dt'}\right) = \frac{\mu_0 c q^2}{16\pi^2} \sin^2\theta' \left(\dot{\beta}'\right)^2 \quad \text{(Eqn 20.:}$$

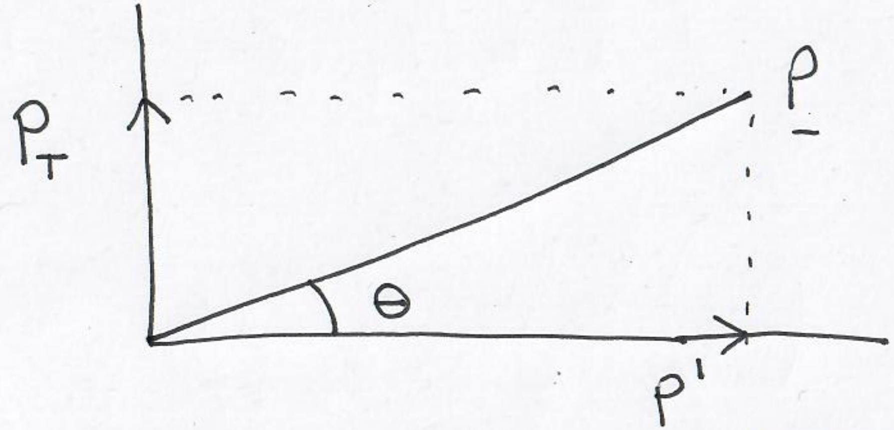
transform this

However, much of the essential physics / maths can be understood by considering Lorentz transformation of a single photon!

Frame: S'



Frame S



(massless) photon has 4-momentum $\underset{\sim}{p} = (p, \underline{p})$

For brevity setting $c=1$. Feel free to replace p with $\frac{E}{c}$ for energy

$$\left.\begin{aligned}
p'_{\parallel} &= p' \cos\theta' & \qquad p_{\parallel} &= p\cos\theta \\
p'_T &= p' \sin\theta' & \qquad p_T &= p\sin\theta \\
p' &= \gamma\left(p - \beta p_{\parallel}\right) = \gamma\left(1 - \beta\cos\theta\right)p \\
p'_{\parallel} &= \gamma\left(p_{\parallel} - \beta p\right) = \gamma\left(\cos\theta - \beta\right)p \\
p'_T &= p_T
\end{aligned}\right\}$$

(Eqns 21.3)

All of the required results follow from the basic equations (21.3)

$$\sin\theta' = \frac{p_T'}{p'} = \frac{p_T}{\gamma(1-\beta\cos\theta)\,p} = \frac{\sin\theta}{\gamma(1-\beta\cos\theta)} \qquad (21.4)$$

$$\cos\theta' = \frac{p_1'}{p'} = \frac{\cos\theta-\beta}{(1-\beta\cos\theta)} \qquad (21.5)$$

$$d(\cos\theta') = \left[\frac{1}{1-\beta\cos\theta} - \frac{(\cos\theta-\beta)(-\beta)}{(1-\beta\cos\theta)^2}\right] d(\cos\theta)$$

$$= \frac{d(\cos\theta)}{(1-\beta\cos\theta)^2}\left(1-\beta^2-\cancel{\beta\cos\theta}+\cancel{\beta\cos\theta}\right)$$

$$\therefore\ d(\cos\theta') = \frac{d(\cos\theta)}{\gamma^2(1-\beta\cos\theta)^2} \qquad (21.6)$$

$$d\Omega' = \sin\theta'\, d\theta'\, d\phi' = -d(\cos\theta')\, d\phi'$$

$\therefore$ using (21.6) and $d\phi' = d\phi$

$$d\Omega' = \frac{-d(\cos\theta)}{\gamma^2(1-\beta\cos\theta)^2}\, d\phi = \frac{d\Omega}{\gamma^2(1-\beta\cos\theta)^2} \qquad (21.7)$$

Considering radiated energy $d\mathcal{E}'$ contained within solid angle $d\Omega'$

$$d\mathcal{E}' = \gamma(1-\beta\cos\theta)\, d\mathcal{E} \qquad (21.8)$$

Note: x' component of momentum carried by radiation within $d\Omega$ is given by $\cos\theta\, d\mathcal{E}$.

We want to evaluate the rate at which energy is radiated by the charge

$$dt' = d\tau = \frac{dt}{\gamma} \quad \text{← time interval in S} \qquad (21.9)$$

↑ proper time

Using (21.7), (21.8), (21.9) and substituting into the left-hand side of Eqn (20.3)

$$\frac{d}{d\Omega'}\left(\frac{d\varepsilon'}{dt'}\right) = \left[\gamma^2(1-\beta\cos\theta)^2 \frac{d}{d\Omega}\right] \frac{\left[\gamma(1-\beta\cos\theta)\, d\varepsilon\right]}{\left[dt/\gamma\right]}$$

$$= \gamma^4(1-\beta\cos\theta)^3 \frac{d}{d\Omega}\left(\frac{d\varepsilon}{dt}\right)$$

$$\therefore \frac{dP'}{d\Omega'} = \gamma^4(1-\beta\cos\theta)^3 \frac{dP}{d\Omega} \qquad (21.10)$$