Lecture 21(c) Angular distribution of radiation for Bremsstrahlung and Synchrotron Radiation

Case 1) Bremsstrahlung

$\underline{\beta} \parallel \underline{\dot{\beta}}$ $\qquad \dot{\beta}' = \gamma^3 \dot{\beta}$

Substitute (21.4), (21.10) into (20.3)

$$\gamma^4 (1-\beta\cos\theta)^3 \frac{dP}{d\Omega} = \frac{\mu_0 c q^2}{16\pi^2} \frac{\sin^2\theta}{\gamma^2(1-\beta\cos\theta)^2} \gamma^6 \dot{\beta}^2$$

$$\therefore \frac{dP}{d\Omega} = \frac{\mu_0 c q^2}{16\pi^2} \frac{\sin^2\theta}{(1-\beta\cos\theta)^5} \dot{\beta}^2 \qquad (21.11)$$

Exercise for you: cross check (21.11) by integrating $\frac{dP}{d\Omega}$ over full solid angle to obtain equation (21.1)

The following integral may be useful $\displaystyle\int_{-1}^{1} \frac{(1-x^2)}{(1-\beta x)^5}\, dx = \frac{4\gamma^6}{3}$

B) Angular Distributions

From Lecture 21 (b)

Transforming $\frac{dP'}{d\Omega'}$ is a bit more complicated!

$$\boxed{\frac{dP'}{d\Omega'} = \frac{d}{d\Omega'}\left(\frac{d\mathcal{E}'}{dt'}\right) = \frac{\mu_0 c q^2}{16\pi^2} \sin^2\theta' \left(\dot{\beta}'\right)^2 \quad \text{(Eqn 20.}}$$

transform this

However, much of the essential physics / maths can be understood by considering Lorentz transformation of a single photon!

We want to evaluate the rate at which energy is radiated by the charge

$$dt' = d\tau = \frac{dt}{\gamma} \quad (21.9)$$

$d\tau$: proper time

dt: time interval in S

Using (21.7), (21.8), (21.9) and substituting into the left-hand side of Eqn (20.3)

$$\frac{d}{d\Omega'}\left(\frac{d\varepsilon'}{dt'}\right) = \left[\gamma^2(1-\beta\cos\theta)^2 \frac{d}{d\Omega}\right] \frac{[\gamma(1-\beta\cos\theta)\,d\varepsilon]}{[dt/\gamma]}$$

$$= \gamma^4(1-\beta\cos\theta)^3 \frac{d}{d\Omega}\left(\frac{d\varepsilon}{dt}\right)$$

From Lecture 21(b)

$$\therefore \boxed{\frac{dP'}{d\Omega'} = \gamma^4(1-\beta\cos\theta)^3 \frac{dP}{d\Omega} \quad (21.10)}$$

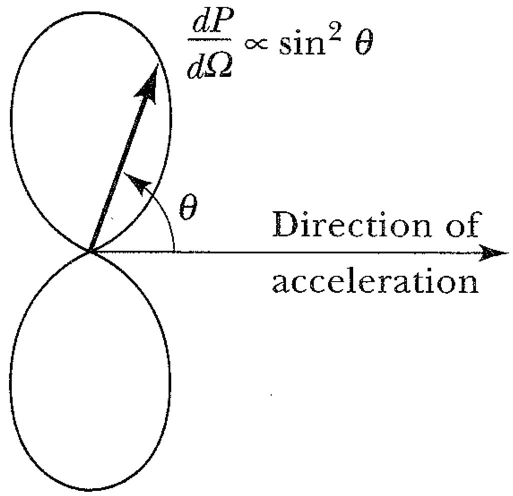


Figure 1: Angular dependence of (Larmor) radiation from a slowly moving accelerating charge $\beta \ll 1$ [1].

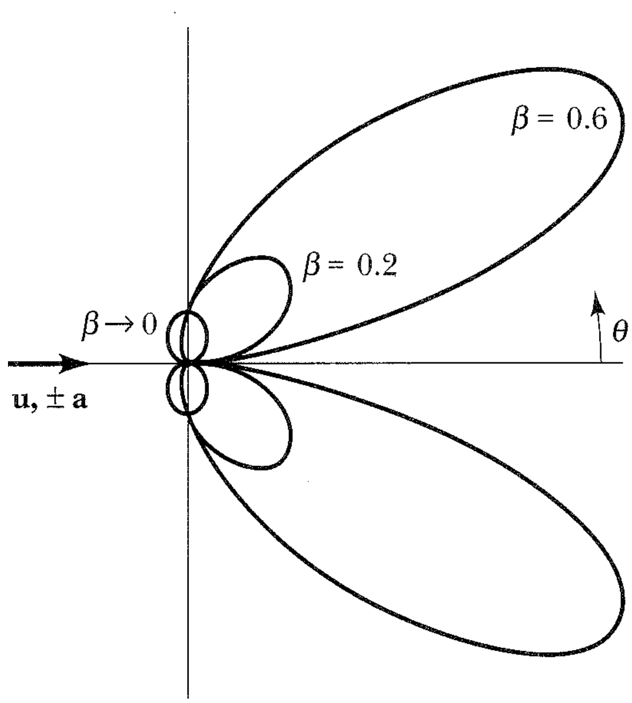


Figure 2: Angular dependence of bremsstrahlung radiation — acceleration is parallel to direction of motion [1].

References

[1] *'Classical Electromagnetic Radiation (3rd edition)'*, M.A.Heald and J.B.Marion, Saunders College Publishing.

N.B. As β increases the radiation is increasingly concentrated in the direction of the velocity

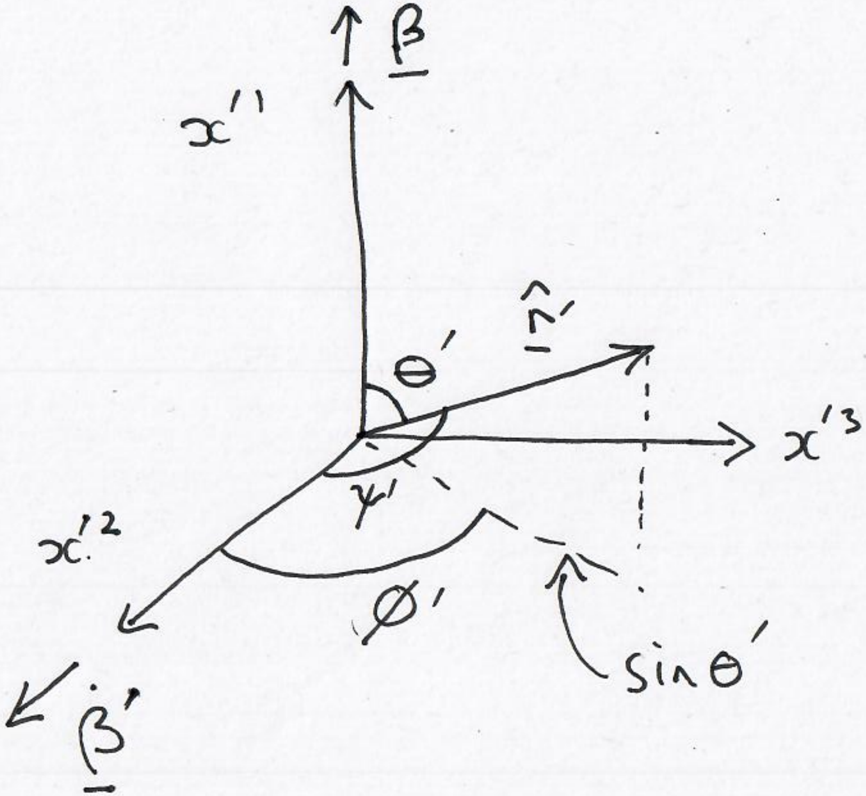
(irrespective of whether $\dot{\underline{\beta}} \cdot \underline{\beta}$ is positive or negative)

Bremsstrahlung has many practical applications in physics:

e.g.,
- X-ray machines
- Particle detectors

Case 2) Synchrotron Radiation

$\underline{\beta} \perp \underline{\dot{\beta}} \qquad \dot{\beta}' = \gamma^2 \dot{\beta}$



Polar coordinate system

θ' relative to x'^1 axis

$\underline{\beta}$ along x'^1 axis

$\underline{\dot{\beta}}'$ along x'^2 axis

ψ' angle between $\hat{\underline{r}}'$ and $\underline{\dot{\beta}}'$

Consider a unit vector $\hat{\underline{r}}' = (1, \theta', \phi')$ that is at an angle ψ' to x'^2 axis

$$\therefore \quad \cos\psi' = x'^2 = \sin\theta' \cos\phi'$$

In S':

$$\sin^2\psi' = 1 - \cos^2\psi' = 1 - \sin^2\theta' \cos^2\phi' \qquad (21.12)$$

Substituting (21.4), (21.10), (21.12) into

$$\frac{dP'}{d\Omega'} = \frac{\mu_0 c q^2}{16\pi^2} \sin^2\psi' \left(\dot{\beta}'\right)^2$$

yields

$$\gamma^4 \left(1 - \beta\cos\theta\right)^3 \frac{dP}{d\Omega} = \frac{\mu_0 c q^2}{16\pi^2}\left(1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2\left(1-\beta\cos\theta\right)^2}\right)\gamma^4\dot{\beta}^2$$

$$\therefore \frac{dP}{d\Omega} = \frac{\mu_0 c q^2}{16\pi^2}\dot{\beta}^2 \frac{1}{\left(1-\beta\cos\theta\right)^3}\left(1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2\left(1-\beta\cos\theta\right)^2}\right) \qquad (21.13)$$

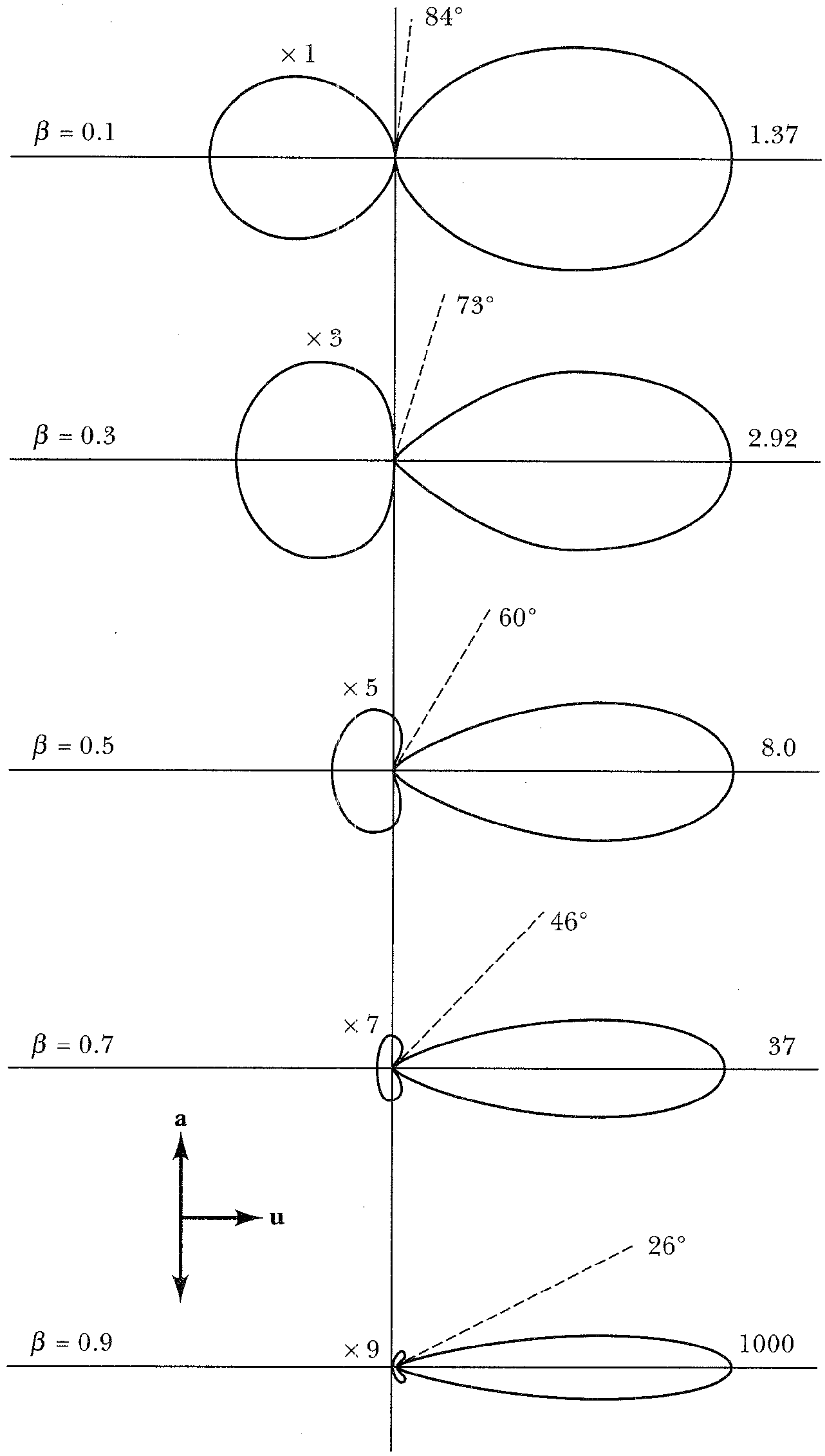


Figure 3: Angular dependence of synchrotron radiation in the plane of the orbit — the acceleration (which is vertical) is perpendicular to the direction of motion (which is to the right) [1].