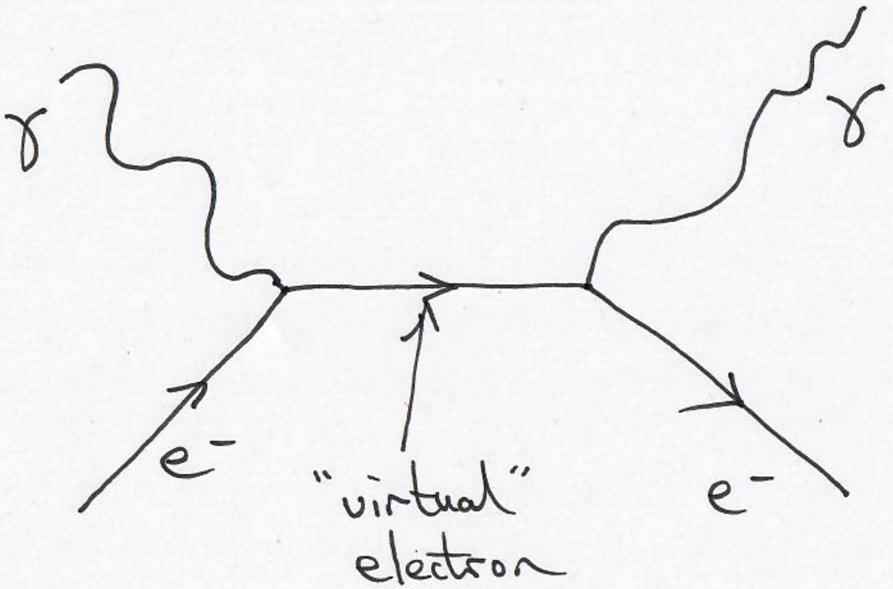
Lecture 22) Two examples of the interaction of e.m. radiation with electrons

A) "relativistic" — Compton scattering

B) "classical" — Thomson scattering.

A) Example of Relativistic Kinematics – Compton Scattering



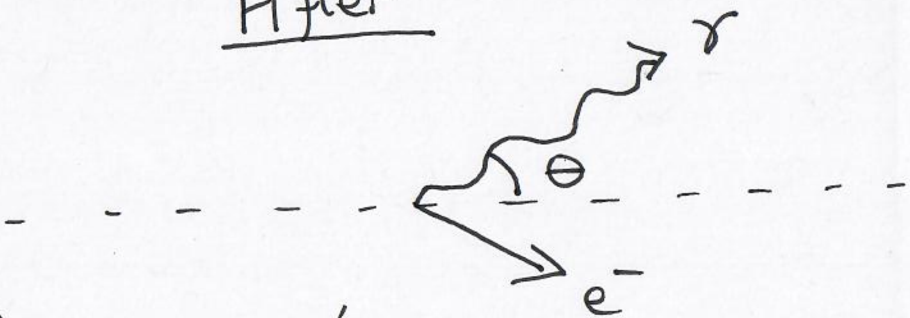
Work with units $c=1$

$$E_\gamma = p_\gamma$$

Before

$$\underset{\sim}{p}_\gamma = (p_\gamma, p_\gamma, 0, 0) \qquad \underset{\sim}{p}_e = (m, 0, 0, 0)$$

After



$$\underset{\sim}{p}'_\gamma = (p'_\gamma, \underline{p}'_\gamma)$$

$$\underset{\sim}{p}'_e = (E'_e, \underline{p}'_e)$$

We shall work out p_γ' as a function of θ

$$\underset{\sim}{p}_\gamma + \underset{\sim}{p}_e = \underset{\sim}{p}_\gamma' + \underset{\sim}{p}_e'$$

$$\left(\underset{\sim}{p}_e'\right)^2 = \left(\underset{\sim}{p}_\gamma + \underset{\sim}{p}_e - \underset{\sim}{p}_\gamma'\right)^2 = \underset{\sim}{p}_\gamma^2 + \underset{\sim}{p}_e^2 + \underset{\sim}{p}_\gamma'^2 + 2\left(\underset{\sim}{p}_\gamma \cdot \underset{\sim}{p}_e - \underset{\sim}{p}_\gamma \cdot \underset{\sim}{p}_\gamma' - \underset{\sim}{p}_e \cdot \underset{\sim}{p}_\gamma'\right)$$

$$m^2 = 0 + m^2 + 0 + 2\left(p_\gamma m - \left[p_\gamma p_\gamma' - \underline{p}_\gamma \cdot \underline{p}_\gamma'\right] - m\, p_\gamma'\right)$$

$$p_\gamma m = p_\gamma' \left[p_\gamma (1 - \cos\theta) + m\right]$$

$$\therefore \quad \boxed{\frac{p_\gamma'}{p_\gamma} = \frac{m}{p_\gamma[1 - \cos\theta] + m}}$$

"The Compton scattering formula"

B) Thomson scattering – example of the use of Larmor Formula

Low energy $\left\{\begin{array}{l}\text{classical}\\ \text{non-QM}\\ \text{non-relativistic}\end{array}\right\}$ limit of Compton scattering

Valid when $\hbar\omega \ll mc^2$

↑ mass of electron.

Consider incoming linearly polarised e.m. wave scattering elastically off of a free electron

E.g.,
$$\underline{E} = \hat{\underline{y}}\, E_0 \cos(\omega t - kx)$$

In radiation fields we found $|B| = |E|/c$

$\therefore$ given $\underline{F} = q(\underline{E} + \underline{v} \times \underline{B}) \sim q\,\underline{E}$

Substitute (19.16) and (19.17) into (19.9)

$$\underline{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{\hat{\underline{R}}}{R^2} + \frac{\hat{\underline{R}} \times (\hat{\underline{R}} \times \dot{\underline{\beta}})}{cR}\right]_{ret}$$

Similarly can show that

$$\boxed{\underline{B} = \frac{1}{c}\left[\hat{\underline{R}}\right]_{ret} \times \underline{E}}$$

Simplified version of the **Liénard-Wiechert** fields valid for the case $\beta \to 0$

From Lecture 19b

Acceleration of electron (charge e)

$$\dot{\beta} c = \frac{eE}{m}$$

Take time averages

$$\langle \dot{\beta}^2 \rangle = \frac{e^2}{m^2 c^2} \langle \underline{E}^2 \rangle = \frac{e^2}{m^2 c^2} \frac{E_0^2}{2}$$

Larmor radiation (eqn 20.3) :

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{\mu_0 c e^2}{16\pi^2} \sin^2\theta \langle \dot{\beta}^2 \rangle$$

$$= \frac{\mu_0}{c} \frac{e^4 E_0^2}{32\pi^2 m^2} \sin^2\theta$$

Note: independent of ω of incoming radiation.

Total scattered power (using equation 20.4)

$$\langle P \rangle = \frac{\mu_0}{c} \frac{e^4 E_0^2}{12\pi m^2}$$

In describing scattering it is useful to use the concept of a cross-section

Here we define the "Thomson cross section" by

$$\sigma_T = \frac{\langle P \rangle}{\langle S \rangle}$$

← The power radiated (or scattered) by a single electron

← Incident power per unit area

Note σ_T has units of area.

$$\langle S \rangle = \frac{1}{\mu_0 c} \langle E^2 \rangle = \frac{E_0^2}{2\mu_0 c}$$

$$\boxed{\sigma_T = \frac{\mu_0^2 e^4}{6\pi m^2}}$$

"Thomson scattering total cross section"

Also useful - but more difficult to imagine is the concept of a "differential cross section"

$$\frac{d\sigma_T}{d\Omega} = \frac{\mu_0^2 e^4}{16\pi^2 m^2} \sin^2\Theta$$

$$= r_c^2 \sin^2\Theta$$

where $r_c = \frac{\mu_0 e^2}{4\pi m}$ is called the "classical electron radius"

Of course we think of electrons as point-like particles (to the limit of our current experimental resolution)!

However, this is a useful concept to descibe scattering of radiation by electrons in this situation.