Exercise for you: Integrate over the entire solid angle in θ and ϕ to obtain the given result.

$$\oint = \frac{q}{\varepsilon_0}$$

From the principle of superposition this result generalizes to multiple point charges q_i enclosed by S

$$\oint = \frac{1}{\varepsilon_0} \sum_i q_i$$

or for a continuous charge we can do a volume integral.

$$\frac{1}{\varepsilon_0} \int_v \rho(\underline{r}') \, d\tau' = \oint = \oint_S \underline{E} \cdot \underline{da} = \int_v \nabla \cdot \underline{E} \, d\tau'$$

where v is the volume enclosed by S

True for all v only if $\nabla \cdot \underline{E} = \rho/\varepsilon_0$