

$$\Phi = \int d\Phi = \frac{q}{4\pi\epsilon_0} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^\pi \sin\theta d\theta}_{-\int_1^{-1} d(\cos\theta) = 2}$$

$$\begin{aligned}\sin\theta d\theta &= -d(\cos\theta) \\ \theta = 0, \cos\theta &= 1 \\ \theta = \pi, \cos\theta &= -1\end{aligned}$$

$$\therefore \Phi = \frac{q}{\epsilon_0}$$

From the principle of superposition this result generalizes to multiple point charges q_i enclosed by S

$$\Phi = \frac{1}{\epsilon_0} \sum_i q_i$$

or for a continuous charge we can do a volume integral.

$$\frac{1}{\epsilon_0} \int_v \rho(\mathbf{r}') d\tau' = \Phi = \oint_s \underline{E} \cdot \underline{da} = \int_v \nabla \cdot \underline{E} d\tau'$$

where v is the volume enclosed by S

True for all v only if $\nabla \cdot \underline{E} = \rho/\epsilon_0$