$$\oint \frac{da}{R} = r^2 \int_0^{2\pi} d\phi \int_0^{\pi} \frac{\sin\theta \, d\theta}{(z^2 + r^2 - 2zr\cos\theta)^{1/2}}$$ $d(\cos\theta) = -\sin\theta \, d$

$$= -r^2 (2\pi)\left(\frac{2}{-2zr}\right)\left[(z^2+r^2-2zr\cos\theta)^{1/2}\right]_0^{\pi}$$

$$= -r^2 (2\pi)\left(\frac{-2}{2zr}\right)\left[(z^2+r^2+2zr)^{1/2} - (z^2+r^2-2zr)^{1/2}\right]$$

$$= -r^2 (2\pi)\left(\frac{-2}{2zr}\right)\left[(z+r) - (z-r)\right]$$

↑ N.B. +ve root since $r < z$

$$= \frac{4\pi r^2}{z} \quad \forall \, r < z$$

Substituting back into Eqn 3.1 above gives

$$V_{av} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{z}$$ as expected for V at distance z from q

By superposition principle this result valid also for collection of charge