In Lecture 3) you were asked to evaluate an integral as a short exercise.

In the answer (shown on the next page) a positive square root was chosen.

In an interactive session we discussed:

(a) The justification for taking the positive, rather than the negative square root

(b) Whether any physical interpretation could be given to the solution obtained if the negative square root were to be taken.

Please refer to the Lecture notes for a diagram and other details of the calculation.

$$\oint \frac{da}{R} = r^2 \int_0^{2\pi} d\phi \int_0^{\pi} \frac{\sin\theta \, d\theta}{(z^2 + r^2 - 2zr\cos\theta)^{1/2}} \qquad d(\cos\theta) = -\sin\theta \, d\theta$$

$$= -r^2 (2\pi)\left(\frac{2}{-2zr}\right)\left[(z^2+r^2-2zr\cos\theta)^{1/2}\right]_0^{\pi}$$

$$= -r^2 (2\pi)\left(\frac{-2}{2zr}\right)\left[(z^2+r^2+2zr)^{1/2} - (z^2+r^2-2zr)^{1/2}\right]$$

$$= -r^2 (2\pi)\left(\frac{-2}{2zr}\right)\left[(z+r) - (z-r)\right] = -r^2 (2\pi)\left(\frac{-2}{2zr}\right)[2r]$$

↑ N.B. +ve root since $r < z$

$$= \frac{4\pi r^2}{z} \quad \forall \, r < z$$

Substituting back into Eqn 3.1 gives

$$V_{av} = \frac{q}{4\pi\varepsilon_0} \frac{1}{z}$$ as expected for V at distance z from q.

By superposition principle this result valid also for collection of charges.

Discussion regarding (a)

In the definite integral the term

* $(z^2 + r^2 + 2zr)^{1/2} = z + r$ represents physically the maximum value that R can take ($\theta = \pi$)
* $(z^2 + r^2 - 2zr)^{1/2} = z - r$ represents physically the minimum value of R ($\theta = 0$). It does make sense physically for this quantity to be positive.

Remember that we specified that $r < z$, which corresponds physically to the charge lying <u>outside</u> the spherical surface.

Discussion regarding (b)

What if we nevertheless chose instead to use $(z^2+r^2-2zr)^{1/2} = r-z$?

In that case we obtain $\oint \frac{da}{R} = 4\pi r$,

which when substituted back into eqn 3.1 gives $V_{av.} = \frac{q}{4\pi\varepsilon_0 r}$, which suggests the following physical interpretation:

* $r-z$ would be positive for $r > z$, which corresponds to the charge lying <u>inside</u> the spherical surface.

* We can generalise the result we proved in Lecture 3) to

$$V_{\substack{\text{average over}\\ \text{spherical surface}\\ \text{of radius } r}} = V_{\substack{\text{at centre}\\ \text{produced by}\\ \text{charges outside}\\ \text{spherical}\\ \text{surface.}}} + \frac{Q_{tot}}{4\pi\varepsilon_0 r},$$

where Q_{tot} is the total charge contained within the sphere. N.B. It does not matter where within the sphere the charge is located.