Lecture 8) Revision of Statics $\Rightarrow$ Electrodynamics

Conservation of Electric Charge

$$\frac{dQ}{dt} = \int_V \frac{\partial \rho}{\partial t}\, d\tau' = -\oint_S \underline{j} \cdot \underline{da} = -\int_V \nabla \cdot \underline{j}\, d\tau'$$

using the divergence theorem.

we can therefore conclude that

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{j} = 0$$

for this relationship to be true for arbitrary choice of volume.

Continuity equation for charge.

Now that $\nabla \cdot \underline{j}$ can be non-zero we need to modify

$$\nabla \times \underline{B} = \mu_0 \underline{j}$$

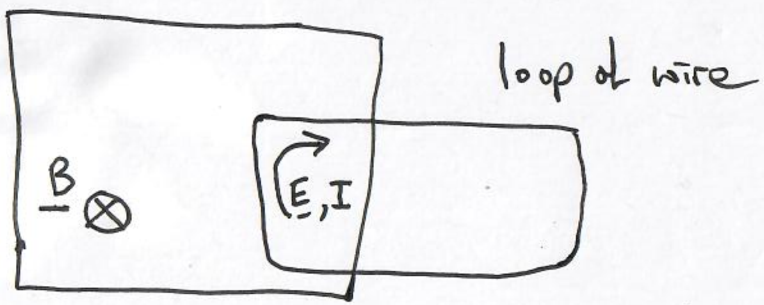
From vector identity (9) we know that

$$0 = \nabla \cdot (\nabla \times \underline{B}) = \mu_0 \underbrace{\left(\nabla \cdot \underline{j} + \frac{\partial \rho}{\partial t} \right)}_{\substack{= \\ 0 \\ \text{from continuity} \\ \text{equation}}} = \mu_0 \left(\nabla \cdot \underline{j} + \varepsilon_0 \frac{\partial}{\partial t} (\nabla \cdot \underline{E}) \right)$$

using Gauss's Law.

$$\therefore \quad \nabla \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

Electromagnetic Induction



Move wire loop to right
Move magnet to left
Decrease B

} All produce the same effect

Induced EMF = − rate of change of magnetic flux through the loop

$\Downarrow$

$$\mathcal{E} = \oint \underline{E} \cdot \underline{dl} = -\frac{d\Phi_m}{dt} = -\int_S \frac{\partial \underline{B}}{\partial t} \cdot \underline{da}$$

$\uparrow$ Apply Stoke's theorem $= \int_S \nabla \times \underline{E} \cdot \underline{da}$

$$\therefore \quad \nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \qquad \underline{\text{Faraday's Law}}$$

(Lenz's Law)

Direction of $\mathcal{E}$, I is so as to oppose change in flux Φ_m through the loop.

Time dependent potentials

Since $\nabla . \underline{B} = 0$ we can still write $\underline{B} = \nabla \times \underline{A}$

However, since $\nabla \times (\nabla V) = 0$, we can no longer write simply $\underline{E} = -\nabla V$
[vector identity (10)]

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} = -\nabla \times \left(\frac{\partial \underline{A}}{\partial t} \right)$$

$\Rightarrow$ New expression for $\underline{E}$ in terms of the potentials

$$\underline{E} = -\nabla V - \frac{\partial \underline{A}}{\partial t}$$

Gauge Transformations

Since $\underline{B} = \nabla \times \underline{A}$ and $\nabla \times (\nabla \psi) = 0$

we can still transform $\underline{A} \rightarrow \underline{A} + \nabla \psi$ without changing $\underline{B}$

require also $V \rightarrow V - \frac{\partial \psi}{\partial t}$ for $\underline{E}$ to remain unchange

Crosscheck

$$\underline{E} \Rightarrow -\nabla\left(V - \frac{\partial \chi}{\partial t}\right) - \frac{\partial}{\partial t}\left(\underline{A} + \nabla \chi\right) = -\nabla V - \frac{\partial \underline{A}}{\partial t}$$

Choice of scalar χ is purely a matter of convenience!

In magnetostatics we chose $\nabla . \underline{A} = 0$, but in electrodynamics we shall make a different choice.