

Since $\rho = \rho(t_{\text{ret}}) = \rho\left(t - \frac{R}{c}\right)$ it will be a solution to the 1-D wave equation

$$\frac{1}{c^2} \frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 \rho}{\partial R^2} \quad (\text{Eqn 9.13})$$

Exercise for you: Verify explicitly that this is the case!

Substituting for (9.13) into (9.12) and then into (9.11) we obtain

$$\nabla^2 V_2 = \frac{1}{4\pi\epsilon_0} \int_{V_2} \frac{1}{R} \frac{1}{c^2} \frac{\partial^2 \rho}{\partial t^2} d\tau' = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left[\underbrace{\frac{1}{4\pi\epsilon_0} \int_{V_2} \frac{\rho(r', t_{\text{ret}})}{R} d\tau'}_{= V(r, t) \text{ as } V_1 \text{ becomes infinitesimally small}} \right]$$

we can write

$$\nabla^2 V_2 = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} \quad (\text{Eqn 9.14})$$

$= V(r, t)$ as V_1 becomes infinitesimally small
 $V_2 \rightarrow V$ the entire space