

Since $\rho = \rho(t_{\text{ret}}) = \rho\left(t - \frac{R}{c}\right)$ we can write :

$$\frac{\partial \rho}{\partial R} = \frac{\partial \rho}{\partial (t - R/c)} \frac{\partial (t - R/c)}{\partial R} = -\frac{1}{c} \frac{\partial \rho}{\partial (t - R/c)}$$

$$\begin{aligned} \frac{\partial^2 \rho}{\partial R^2} &= \frac{\partial}{\partial R} \left(\frac{\partial \rho}{\partial R} \right) = \frac{\partial}{\partial R} \left(-\frac{1}{c} \frac{\partial \rho}{\partial (t - R/c)} \right) \\ &= -\frac{1}{c} \frac{\partial^2 \rho}{\partial (t - R/c)^2} \frac{\partial (t - R/c)}{\partial R} = \frac{1}{c^2} \frac{\partial^2 \rho}{\partial (t - R/c)^2} \end{aligned}$$

Similarly,

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 \rho}{\partial (t - R/c)^2}$$

$\therefore \rho$ will be a solution to the 1-dimensional wave equation

$$\frac{1}{c^2} \frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2 \rho}{\partial R^2}$$

q.e.d.