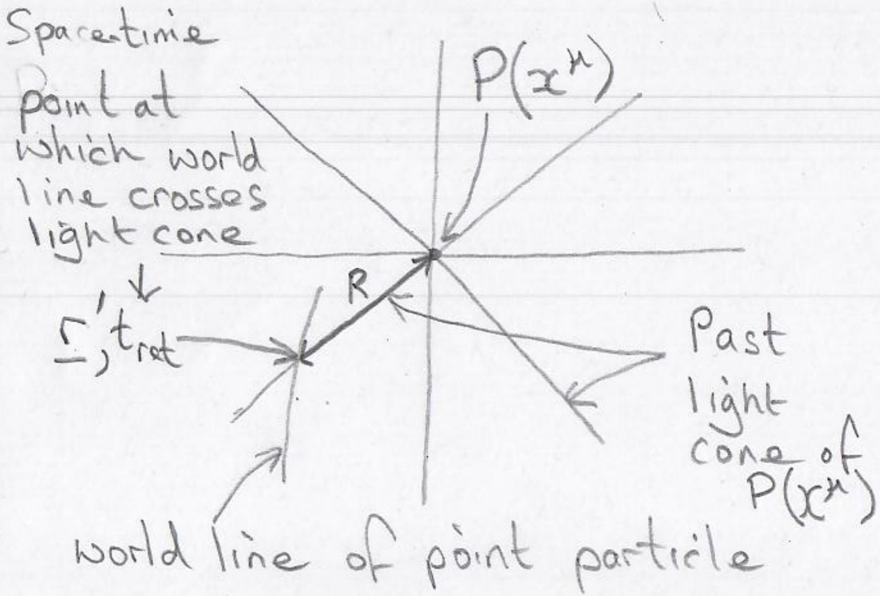
A note on the definition of $[\dot{\beta}]_{ret}$

Physically, we want to define $[\underline{\beta}]_{ret}$ and $[\dot{\underline{\beta}}]_{ret}$ to be the velocity (in units of c) and acceleration of our point particle, as measured in the frame S. These quantities should be measured at the "retarded time" $t_{ret} = t - \frac{R}{c}$ and not the "current time" t.



If particle is moving, then in general R changes with time and therefore

$$\frac{d}{dt_{ret}} = \frac{\partial}{\partial t_{ret}} + \frac{dx'^i}{dt_{ret}} \frac{\partial}{\partial x'^i} \neq \frac{\partial}{\partial t_{ret}}$$

(Eqn A)

The question then arises as to whether we should use $\frac{d}{dt_{ret}}$ or $\frac{\partial}{\partial t_{ret}}$ to define $\underline{\beta}$ and $\dot{\underline{\beta}}$?

As we can see from eqn(A), $\frac{d}{dt_{ret}}$ includes the effect of changes in R as well as the time interval in S. This would be what we would need to study the apparent behaviour of the particle, as inferred from the arrival of light signals at $P(x^{\mu})$! As we found in the optional lecture in week 4 this is an interesting subject, but it is not what we need here

$\therefore$ We <u>define</u> $\underline{\beta} \equiv \frac{\partial \underline{r}'}{\partial t_{ret}}$, $\dot{\underline{\beta}} \equiv \frac{\partial \underline{\beta}}{\partial t_{ret}}$, which depend in a simple fashion on time intervals, as measured by a clock in frame S.